SUPPLEMENTARY APPENDIX

A Heterogeneous supplier costs: Continuous case

In the following, we generalize our analysis to the case of a continuous distribution of supplier costs. Specifically, we assume that the cumulative distribution function $G(c_m)$ is continuously differentiable, and denote by $g(c_m)$ the corresponding probability density function with support $[\underline{c_m}, \overline{c_m}]$.

Let $\widetilde{c_m}$ denote the cost realization of the firm's current supplier, and $\widetilde{\pi_H^i}$ is the firm's payoff at this cost realization given the contract structure $i = \{N, RC\}$. In order to derive whether the firm starts re-matching or not in period 0, where $c_m^0 = \widetilde{c_m}$, the same logic as in the main text applies. To re-match the current supplier, it must be the case that the expected gains must be larger than the continuation payoffs with the current match:

$$E[\pi_H^{RM,i} \mid c_m^0 = \widetilde{c_m}] > \frac{\widetilde{\pi_H^i}}{1-\delta_H}, \tag{24}$$

where, as before, the two cases of impatient and patient agents must be distinguished that ultimately lead to the contract structure i once the firm has stopped searching.

The term $E[\pi_H^{RM,i} \mid c_m^0 = \widetilde{c_m}]$ can be calculated as follows:

$$E[\pi_H^{RM,i} \mid c_m^0 = \widetilde{c_m}] = \widetilde{\pi_H^N} - F + \frac{\delta_H}{1-\delta_H} E[\pi_H^i]$$

Note, that (24) has to be re-evaluated for every supplier encountered in the re-matching procedure.

Analogously as in Section 4, from (24) we can derive the fixed cost thresholds $\overline{F}^k(\widetilde{c_m})$, where $k = 1$ for impatient and $k = 2$ for patient agents, respectively. As before, the thresholds are defined in a way such that for all $F < \overline{F}^k(\widetilde{c_m})$ re-matching is incentive compatible, while otherwise it is not. We get the following thresholds:

$$\begin{aligned} \overline{F}^1(\widetilde{c_m}) &= \tfrac{\delta_H}{1-\delta_H}\left[E[\pi_H^N] - \widetilde{\pi_H^N}\right] \\ \overline{F}^2(\widetilde{c_m}) &= \widetilde{\pi_H^N} + \tfrac{1}{1-\delta_H}\left[\delta_H E[\pi_H^{RC}] - \widetilde{\pi_H^{RC}}\right] \end{aligned}$$

As in the main text, for the case where a RC is feasible (i.e. $\widetilde{\delta} \geq \underline{\delta}$) we must rule out any "cheat-and-run" (CAR) incentives of the firm, i.e. that it repeatedly deviates from the RC and re-matches to a new supplier. For this, search costs must not be too small and it must hold:

$$\frac{\widetilde{\pi_H^{RC}}}{1-\delta_H} \geq E[\pi_H^{CAR} \mid c_m^0 = \widetilde{c_m}],$$

where $E[\pi_H^{CAR} \mid c_m^0 = \widetilde{c_m}] = \widetilde{\pi_H^D} - F + \frac{\delta_H}{1-\delta_H}(E[\pi_H^D] - F)$ is the firm's off-equilibrium payoff when it always deviates from the RC. For CAR never to occur a sufficient condition is $E[\pi_H^{RM,RC} \mid c_m^0 = \widetilde{c_m}] > E[\pi_H^{CAR} \mid c_m^0 = \widetilde{c_m}]$, or:

$$F > \frac{1-\delta_H}{\delta_H}\left(\widetilde{\pi_H^D} - \widetilde{\pi_H^N}\right) + E[\pi_H^D] - E[\pi_H^{RC}] \equiv \widetilde{F}(\widetilde{c_m}).$$

By construction, the condition also guarantees that $\overline{F}^2(\widetilde{c_m}) > \widetilde{F}(\widetilde{c_m})$ for all $\widetilde{c_m}$. This gives the following result:

Proposition 4.

Suppose the headquarter is initially matched to a supplier with costs $c_m^0 = \widetilde{c_m}$.

a) Impatient agents ($\widetilde{\delta} < \underline{\delta}$): Consider any $F > 0$. Impatient agents start re-matching if $F < \overline{F}^1(\widetilde{c_m})$ and continue re-matching until, in period t, they find a supplier with costs c_m^{t+1} for which $F \geq \overline{F}^1(c_m^{t+1})$. The firm will engage in a LTC with any supplier for which $F \geq \overline{F}^1(c_m^{t+1})$ holds. The RC can never be implemented.

b) Patient agents ($\widetilde{\delta} \geq \underline{\delta}$): Consider any $F > \widetilde{F}(\widetilde{c_m})$. Patient agents start re-matching if $F < \overline{F}^2(\widetilde{c_m})$ and continue until, in period t, they find a supplier with costs c_m^{t+1} for which $F \geq \overline{F}^2(c_m^{t+1})$. The RC forms with any supplier for which $F \geq \overline{F}^2(c_m^{t+1})$ holds. Otherwise, the RC cannot be implemented.

Notice, that for both, patient and impatient suppliers, the respective search cost threshold $\overline{F}^k$ decreases in the cost level of the current supplier, $\widetilde{c_m}$. Thus, for given search costs it follows necessarily that the re-matching process stops at some level of supplier efficiency and a LTC is established. Proposition 4 confirms, that the logic of Proposition 2 extends to the case where the distribution of supplier costs is continuous.

B Separated search and re-matching

In this Appendix we propose an alternative specification for the firm's decision of supplier re-matching. Contrary to the baseline model, the firm can now separately decide whether or not she wants to re-match with a supplier that she encounters during the search process. In particular, we modify the re-matching stage as follows:

7. **Re-matching stage**: H can decide to search for a new supplier. When deciding to search, she incurs a publicly known fixed cost $F > 0$. Let c_m^t be the unit cost level of her current supplier, and c_m^{t+1} the unit cost of the new supplier that she has encountered during her search. The cost c_m^{t+1} is randomly drawn from the distribution function $G(c_m)$ which is i.i.d. over periods. If the cost

draw is such that $c_m^{t+1} < c_m^t$, the headquarter re-matches and continues the game with the new supplier. If $c_m^{t+1} \geq c_m^t$ she keeps her previous supplier.

Our assumption is thus that the headquarter can observe the efficiency level c_m^{t+1} of the candidate supplier, and will only re-match if he is more efficient than her current partner. In the following, we will first discuss the case where supplier costs are drawn from a two-point distribution as in the main text and then extend the analysis to the continuous case.

B.1 Two-point distribution of supplier costs

Analogously to the baseline model the search decision has two dimensions:

1. if the initial supplier is a high-cost type ($c_m^0 = c_m^h$), does the firm start searching?
2. if the initial supplier is a low-cost type ($c_m^0 = c_m^l$), or if the firm has found a low-cost type during her search, does she stop searching and stick with that partner?

For the *case of impatient agents* ($\widetilde{\delta} < \underline{\delta}$) the analysis remains the same as in the baseline model. That is, if the firm finds (or is initially matched with) a low-cost supplier, she will stick to that partner since further costly search makes no sense. When the firm is still matched with a high-cost supplier and if search costs are low enough, the firm starts searching. Search stops once she finds a low-cost supplier, but this can require several periods. The only difference to the baseline model is that, during this search period, the firm now has the option to stick with her initial high-cost supplier, rather than switching to another high-cost supplier in every round. However, in both scenarios we have identical (Nash) investments and (Nash) payoffs in the respective stage game round, hence the formal analysis from the baseline model remains unchanged. The search condition $F < \overline{F}^1$ from (**??**) therefore still applies.

For the *case of patient agents* ($\widetilde{\delta} > \underline{\delta}$), the analysis becomes more intricate. First, consider the second aspect. As before, "cheat-and-run" (CAR) can be ruled out by ensuring that the search process stops whenever the firm is matched to a low-cost supplier:

$$\frac{1}{1-\delta_H}\pi_H^{RC,l} \geq E[\pi_H^{CAR} \mid c_m^0 = c_m^l] \tag{25}$$

Compared to (**??**), the expected "cheat-and-run"-payoffs have to be slightly modified for the case where search and re-matching are two separate decisions. They can be formalized by the following program:

$$V_0 = \pi_H^{D,l} - F + \delta_H P V_0 + \delta_H (1-P) V_1, \qquad V_1 = z - F + \delta_H P V_0 + \delta_H (1-P) V_1,$$

where $z = \max\{\pi_H^{N,l}, \pi_H^{D,h}\}$. Note that in the baseline case we had $z = \pi_H^{D,h}$. Now, if the firm encounters a high-cost supplier during her search, depending on the difference in unit costs c_m^h and c_m^l, it may be better to stick to the current low-cost partner and play Nash with him, rather than to re-match and then cheat on the high-cost supplier. As a consequence:

$$E[\pi_H^{CAR} \mid c_m^0 = c_m^l] = \frac{1}{1-\delta_H}\left[(1-\delta_H(1-P))\pi_H^{D,l} + \delta_H(1-P)z - F\right].$$

Using the equivalent steps from the baseline model, from (25) we can derive a lower threshold on search costs, $\widetilde{F}'$, where $F > \widetilde{F}'$ rules out "cheat-and-run" behaviour.

Turning to the first question, search may thus only occur if the initial supplier is a high-cost type, $c_m^0 = c_m^h$, and as before it will actually occur if search costs F are low enough. In particular, and equivalently to (**??**), the expected payoff when engaging in search must be higher than the continuation payoff with the initial high-cost supplier, i.e.,

$$E[\pi_H^{search,RC} \mid c_m^0 = c_m^h] > \frac{1}{1-\delta_H}\pi_H^{RC,h}. \tag{26}$$

If that condition is violated and the firm decides not to search, she forms a RC with her initial high-cost supplier which is sustainable since $\widetilde{\delta} > \underline{\delta}$. If condition (26) is satisfied, the firm starts searching. Once she has found a low-cost supplier, search stops and she forms a long-term RC collaboration with that partner as shown before. But in every search round, the firm encounters a high-cost supplier with probability $(1-P)$, so it may take several periods before the once-and-for-all supplier turnover actually takes places.

Separating the firm's decisions of supplier search and re-matching introduces the possibility to further distinguish the patient agents into two groups that behave differently during the periods of supplier search. In the following we show that for a subset of very patient agents it can be incentive compatible to engage in a RC with the initial high-cost supplier during the search periods, despite the ongoing search for a better partner. Specifically, the RC is better for the firm than Nash play with M_0 if

$$\begin{aligned}\pi_H^{RC,h} \quad &+ \quad \left[(1-P)\pi_H^{RC,h} + P\pi_H^{RC,l}\right] \cdot \left(\delta_H + \delta_H^2(1-P) + \delta_H^3(1-P)^2 + \dots\right) \\ &> \quad \pi_H^{D,h} + \left[(1-P)\pi_H^{N,h} + P\pi_H^{RC,l}\right] \cdot \left(\delta_H + \delta_H^2(1-P) + \delta_H^3(1-P)^2 + \dots\right),\end{aligned}$$

and for M_0 the RC is incentive compatible if

$$\begin{aligned}\pi_M^{RC,h} \quad &+ \quad \pi_M^{RC,h}\left(\delta_M + \delta_M^2(1-P) + \delta_M^3(1-P)^2 + \dots\right) \\ &> \quad \pi_M^{D,h} + (1-P)\pi_M^{N,h}\left(\delta_M + \delta_M^2(1-P) + \delta_M^3(1-P)^2 + \dots\right).\end{aligned}$$

These incentive compatibility constraints thus boil down to

$$\frac{1}{1-\delta_i(1-P)}\pi_i^{RC,h} > \pi_i^{D,h} + \frac{\delta_i(1-P)}{1-\delta_i(1-P)}\pi_i^{N,h} \qquad \text{for} \quad i = H, M,$$

which are similar to (IC-H) and (IC-M) from above, but capture the replacement probability P in every period. A temporary RC with the high-cost initial supplier until replacement is thus optimal if $\widetilde{\delta} \geq \frac{1}{1-P}\frac{\pi^{JFB}-\pi_H^D-\pi_M^D}{\pi_H^N+\pi_M^N-\pi_H^D-\pi_M^D} = \underline{\delta}/(1-P) > \underline{\delta}$, where $\underline{\delta}$ is the previously derived critical discount factor given in (??). Put differently, very patient agents with an average discount factor above $\underline{\delta}/(1-P)$ would always form a RC, both with the initial high-cost and with the final low-cost supplier. By contrast, mildly patient agents with $\underline{\delta} < \widetilde{\delta} < \underline{\delta}/(1-P)$ only form the RC once they have found their final low-cost supplier, but not in the temporary search phase with the high-cost supplier.

Having distinguished the very patient and the mildly patient agents, we can now complete the model extension and derive the critical search cost levels for the very patient agents (the ones for mildly patient agents are the same as in the baseline model and described by $\overline{F}^2$). For the very patient agents who always engage in RCs ($\widetilde{\delta} > \underline{\delta}/(1-P)$), the search decision can be formalized as

$$V_0 = \pi_H^{RC,h} - F + \delta_H V_1, \qquad V_1 = \frac{1}{1-\delta_H(1-P)}\left((1-P)(\pi_H^{RC,h} - F) + \frac{P}{1-\delta_H}\pi_H^{RC,l}\right),$$

which yields these expected profits

$$E[\pi_H^{search,RCstrong} \mid c_m^0 = c_m^h] = \frac{1}{1-\delta_H(1-P)}\left[\pi_H^{RC,h} - F + \frac{\delta_H P}{1-\delta_H}\pi_H^{RC,l}\right] \tag{27}$$

Plugging (27) into (26) and rearranging we obtain the critical search cost level $\overline{F}^{2^*}$ for the very patient agents case:

$$F < \frac{\delta_H P}{1-\delta_H}\left[\pi_H^{RC,l} - \pi_H^{RC,h}\right] \equiv \overline{F}^{2^*}. \tag{28}$$

Comparing (??), (??) and (28), it can be verified that $\overline{F}^{2^*} > \overline{F}^2$ always holds. Moreover, as $\overline{F}^1$ and $\overline{F}^2$, also $\overline{F}^{2^*}$ is increasing in δ_H (for given δ_M). Finally, we can make a similar consistency argument as in Appendix B to guarantee $\overline{F}^2 > \widetilde{F}'$ which in turn implies $\overline{F}^{2^*} > \widetilde{F}'$ since $\overline{F}^{2^*} > \overline{F}^2$.

We summarize our results under separated search and re-matching in the following Propositions 5 and 6, which are analogous to Propositions 2 and 3 from the main text and now incorporate the distinction of mildly patient and very patient agents.

Proposition 5. *a) Suppose the headquarter is initially matched with a low-cost supplier ($c_m^0 = c_m^l$). Patient agents (with $\widetilde{\delta} > \underline{\delta}$) will collaborate with that supplier* *forever in a relational contract (RC) agreement with $\{h^*, m^*\}$ and $B^*(\delta)$, assuming that re-matching costs are not too low ($F > \widetilde{F}'$). Impatient agents (with $\widetilde{\delta} < \underline{\delta}$) will form a long-term collaboration of repeated Nash bargainings with that supplier.*

b) With $c_m^0 = c_m^h$, impatient agents with $\widetilde{\delta} < \underline{\delta}$ search if $\widetilde{F} < F < \overline{F}^1$ and continue searching until they find a low-cost supplier. The RC can never be implemented.

c) With $c_m^0 = c_m^h$, mildly patient agents with $\underline{\delta} < \widetilde{\delta} < \underline{\delta}/(1-P)$ search if $\widetilde{F} < F < \overline{F}^2$ and continue searching until they find a low-cost supplier. The RC forms with the final low-cost supplier, but not with the initial high-cost supplier.

d) With $c_m^0 = c_m^h$, very patient agents with $\widetilde{\delta} > \underline{\delta}/(1-P)$ search if $\widetilde{F} < F < \overline{F}^{2^*}$ and continue searching until they find a low-cost supplier. The RC forms both with the final low-cost supplier, and with the initial high-cost supplier during the search period.

Moreover, we state the following result referring to the impacts of a mean-preserving spread (MPS) in the distribution of supplier costs:

Proposition 6. *A mean-preserving spread in the distribution of supplier costs (a larger difference between c_m^l and c_m^h at constant P) increases the critical search cost levels $\overline{F}^1$, $\overline{F}^2$ and $\overline{F}^{2^*}$, and thereby expands the parameter range where the headquarter engages in search.*

We illustrate the changes that result from separating the decisions of search and re-matching in Figure 3, which is comparable to Figure 2. The dotted horizontal line at $\underline{\delta}/(1-P)$ indicates the critical discount factor above which agents are very patient and an RC is formed also during the search for a better partner. We label this a *short-term collaboration* (STC), since it is neither a one-shot nor a truly long-term interaction. In the range between the dotted and the solid horizontal line (at $\underline{\delta}$) agents are mildly patient, and play Nash during the STC-phase and only turn to the RC once the LTC is launched. This is the causal effect *LTC → RC* studied in the main text. Finally, below the solid line agents are impatient and always play Nash.

A MPS on the distribution of supplier costs has the same effect on $\overline{F}^{2^*}$ as it has on $\overline{F}^2$ and $\overline{F}^1$ and shifts the $\overline{F}^{2^*}$-function outwards. In total, also when separating the search decision from the re-matching decision, a MPS on $G(c_m)$ unambiguously increases the firm's propensity to search. For the mildly patient agents, this indirectly affects contractual structures because they only offer a RC once they have found their ultimate low-cost supplier. Yet, as in the baseline model, the MPS does not directly affect contractual structures since $\underline{\delta}$ and $\underline{\delta}/(1-P)$ are both independent of c_m. As for Figure 2 in the main text, the concave form of the $\overline{F}^k$-functions obtains whenever we consider changes is δ_H while holding δ_M constant.

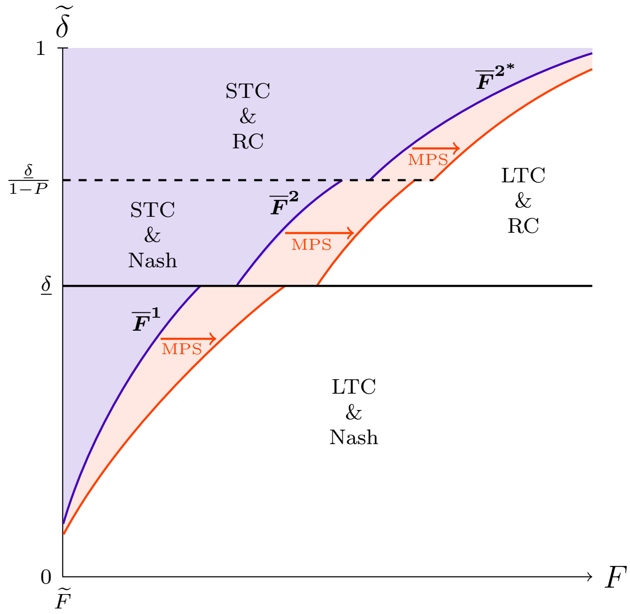

Figure 3: Contractual and search/re-matching decision ($c_m^0 = c_m^h$)

B.2 Continuous distribution of supplier costs

In the following, we extend the analysis to the case of a continuous distribution of supplier costs. These get drawn from a continuously differentiable cdf $G(c_m)$, and we denote by $g(c_m)$ the corresponding pdf with support $[\underline{c_m}, \overline{c_m}]$. The analysis is tightly related to that of Supplementary Appendix A, where search and re-matching were no separable decisions. However, as already pointed out above, the separability of search and re-matching generates a class of very patient agents that engage in a RC while being on the search for a better partner.

Let $\widetilde{c_m}$ denote the cost realization of the firm's current supplier, and let $\widetilde{\pi_H^i}$ be the firm's payoff at this cost realization given the contract structure $i = \{N, RC\}$. In order that the firm starts searching it must be the case that the expected gains are larger than the continuation payoffs with the current match:

$$E[\pi_H^{RM,j} \mid c_m^0 = \widetilde{c_m}] > \frac{\widetilde{\pi_H^i}}{1 - \delta_H}, \tag{29}$$

where the three cases of agents, $j \in \{\text{impatient, mildly patient, very patient}\}$, must be distinguished. Ultimately, once search has stopped impatient agents will engage in contract structure $i = N$ and both types of patient agents in $i = RC$. The term

$E[\pi_H^{RM,j} \mid c_m^0 = \widetilde{c_m}]$ can be calculated as follows:

$$
\begin{aligned}
E[\pi_H^{RM,\text{ impatient}} \mid c_m^0 = \widetilde{c_m}] &= \frac{1}{1-\delta_H(1-G(\widetilde{c_m}))}\left[\widetilde{\pi_H^N} - F + \frac{\delta_H G(\widetilde{c_m})}{1-\delta_H} E[\pi_H^N \mid c_m < \widetilde{c_m}]\right] \\
E[\pi_H^{RM,\text{ mildly patient}} \mid c_m^0 = \widetilde{c_m}] &= \frac{1}{1-\delta_H(1-G(\widetilde{c_m}))}\left[\widetilde{\pi_H^N} - F + \frac{\delta_H G(\widetilde{c_m})}{1-\delta_H} E[\pi_H^{RC} \mid c_m < \widetilde{c_m}]\right] \\
E[\pi_H^{RM,\text{ very patient}} \mid c_m^0 = \widetilde{c_m}] &= \frac{1}{1-\delta_H(1-G(\widetilde{c_m}))}\left[\widetilde{\pi_H^{RC}} - F + \frac{\delta_H G(\widetilde{c_m})}{1-\delta_H} E[\pi_H^{RC} \mid c_m < \widetilde{c_m}]\right]
\end{aligned}
$$

Note, that (29) has to be re-evaluated for every supplier that the firm encounters in the re-matching procedure. By rearranging (29), we can obtain the critical search costs $\overline{F}^k$ for all the three patience levels that now each depend on the current cost realization $\widetilde{c_m}$. Using the same notation as in the previous subsection we get:

$$
\begin{aligned}
\overline{F}^1(\widetilde{c_m}) &= \frac{\delta_H G(\widetilde{c_m})}{1-\delta_H}\left[E[\pi_H^N \mid c_m < \widetilde{c_m}] - \widetilde{\pi_H^N}\right] \\
\overline{F}^2(\widetilde{c_m}) &= -\left(\widetilde{\pi_H^{RC}} - \widetilde{\pi_H^N}\right) + \frac{\delta_H G(\widetilde{c_m})}{1-\delta_H}\left[E[\pi_H^{RC} \mid c_m < \widetilde{c_m}] - \widetilde{\pi_H^{RC}}\right] \\
\overline{F}^{2^*}(\widetilde{c_m}) &= \frac{\delta_H G(\widetilde{c_m})}{1-\delta_H}\left[E[\pi_H^{RC} \mid c_m < \widetilde{c_m}] - \widetilde{\pi_H^{RC}}\right]
\end{aligned}
$$

By using the same steps as in Supplementary Appendix A, we can derive a lower bound on search costs $\widetilde{F}(\widetilde{c_m})$ under which CAR can be ruled out. Because we have $\overline{F}^{2^*}(\widetilde{c_m}) > \overline{F}^2(\widetilde{c_m})$ for all $\widetilde{c_m}$ it is sufficient to construct the lower bound for mildly patient agents in order to rule out CAR for all agents. Hence we can formulate the following proposition:

Proposition 7.

Suppose the headquarter is initially matched to a supplier with costs $c_m^0 = \widetilde{c_m}$.

a) Impatient agents ($\widetilde{\delta} < \underline{\delta}$): Consider any $F > 0$. Impatient agents start searching if $F < \overline{F}^1(\widetilde{c_m})$ and continue searching until, in period t, they find a supplier with costs c_m^{t+1} for which $F \geq \overline{F}^1(c_m^{t+1})$. The firm will engage in a LTC with any supplier for which $F \geq \overline{F}^1(c_m^{t+1})$ holds. The RC can never be implemented.

b) Mildly patient agents ($\underline{\delta}/(1-P) > \widetilde{\delta} \geq \underline{\delta}$): Consider any $F > \widetilde{F}(\widetilde{c_m})$. Mildly patient agents start searching if $F < \overline{F}^2(\widetilde{c_m})$ and continue until, in period t, they find a supplier with costs c_m^{t+1} for which $F \geq \overline{F}^2(c_m^{t+1})$. The RC forms with any supplier for which $F \geq \overline{F}^2(c_m^{t+1})$ holds. Otherwise, the RC cannot be implemented.

b) Very patient agents ($\widetilde{\delta} \geq \underline{\delta}/(1-P)$): Consider any $F > \widetilde{F}(\widetilde{c_m})$. Very patient agents start searching if $F < \overline{F}^{2^*}(\widetilde{c_m})$ and continue until, in period t, they find a supplier with costs c_m^{t+1} for which $F \geq \overline{F}^{2^*}(c_m^{t+1})$. The RC forms with any supplier, in STCs and LTCs.

Moreover, the MPS result from above can be extended to continuous distributions:

Proposition 8. *A mean-preserving spread on the continuous distribution of supplier costs increases the critical search cost levels* $\overline{F}^1$*,* $\overline{F}^2$ *and* $\overline{F}^{2^*}$*, and thereby expands the parameter range where the headquarter engages in search.*

Proof. Suppose $\hat{G}(c_m)$ is a MPS on $G(c_m)$ and denote by $\hat{F}^k$, $k \in \{1, 2, 2^*\}$, the associated search cost thresholds. Then in order to show that the Proposition holds it is sufficient to show that, for all k:

$$\hat{F}^k - \overline{F}^k > 0.$$

Plugging in the expression from above this can be simplified to:

$$\hat{G}(\widetilde{c_m}) \left[E_{\hat{G}}[\pi_H^i \mid c_m < \widetilde{c_m}] - \widetilde{\pi_H^i} \right] - G(\widetilde{c_m}) \left[E_G[\pi_H^i \mid c_m < \widetilde{c_m}] - \widetilde{\pi_H^i} \right] > 0,$$

where $i \in \{N, RC\}$. Integrating the conditional expected values by parts, we can simplify this expression to:

$$\int_{\underline{c_m}}^{\widetilde{c_m}} \frac{\partial \pi_H^i}{\partial c_m} G(c_m) \mathrm{d}c_m > \int_{\underline{c_m}}^{\widetilde{c_m}} \frac{\partial \pi_H^i}{\partial c_m} \hat{G}(c_m) \mathrm{d}c_m \tag{30}$$

Observing that $\frac{\partial \pi_H^i}{\partial c_m} < 0$, we can conclude that the Proposition holds. □

C Heterogeneous discount factors: Continuous case

In this Appendix, we analyze the question of supplier search and re-matching where the supplier's discount factor δ_M is heterogeneous and drawn from the continuous interval $(0, 1)$. We focus on the case of stationary bonus payments which ensures that the firm will set the same bonus in every period such that the supplier's IC constraint exactly binds. After every round of output realization the firm can decide whether to re-match with a new supplier. Search and re-matching cannot be separated. For this Appendix, suppliers are homogeneous w.r.t. their costs c_m. The re-matching stage of the game is adjusted as follows.

7. **Re-matching stage**: H can pay a publicly known fixed cost $F > 0$ and re-match to a new supplier. Let δ_M^t be the discount factor of her current supplier, and δ_M^{t+1} the discount factor of the new supplier that she has encountered. This discount factor δ_M^{t+1} is randomly drawn from the distribution function $g(\delta_M)$ and is perfectly observable to the headquarter.

We assume that the cumulative distribution function $G(\delta_M)$ is continuously differentiable on its entire support, $\delta_M \in (0, 1)$. Clearly, depending on search costs F

and the discount factor δ_M^t of its current match, the firm will have an incentive to search for a more patient supplier. We assume δ_H to take a fixed value and to make the exposition interesting we restrain the values of it on the interval $\delta_H \in (\underline{\delta_H}, \overline{\delta_H})$ introduced in the following Corollary to Proposition 1. This restriction guarantees that there will always be some impatient suppliers which which a RC will be impossible, while for patient ones it will be possible. Given that $\delta_H < \underline{\delta_H}$ the RC will always fail no matter how patient the supplier is. On the other hand, if $\delta_H > \overline{\delta_H}$ the RC will always work no matter how impatient the supplier is.

Corollary 1. *Suppose that $\delta_H \in (\underline{\delta_H}, \overline{\delta_H})$, where $\underline{\delta_H} = \max\left\{0, \frac{\pi_H^D + \pi_M^N - \pi^{JFB}}{\pi_H^D - \pi_H^N}\right\}$ and $\overline{\delta_H} = \min\left\{1, \frac{\pi_H^D + \pi_M^D - \pi^{JFB}}{\pi_H^D - \pi_H^N}\right\}$. Then there exists $\underline{\delta_M} \in (0,1)$ such that Nash will be played with all suppliers for which $\delta_M < \underline{\delta_M}$, and a RC with (h^*, m^*) is implementable for all $\delta_M \geq \underline{\delta_M}$.*

Proof. First, consider $\underline{\delta_H}$. If δ_H is "too small", then it will never be possible to start a relational contract no matter how patient the supplier is, i.e. even when $\delta_M \to 1$. Suppose that indeed $\delta_M = 1$ and plug it into (1) which thus can be simplified to $\delta_H \leq \frac{\pi_H^D + \pi_M^N - \pi^{JFB}}{\pi_H^D - \pi_H^N}$. Whenever this condition is satisfied, a RC will not be possible no matter how patient the supplier is.

On the other hand, consider $\overline{\delta_H}$. If δ_H gets "too large", then it will always be possible to start a relational contract no matter how impatient the supplier is, i.e. even when $\delta_M \to 0$. For $\delta_M = 0$, (1) can be rearranged to $\delta_H \geq \frac{\pi_H^D + \pi_M^D - \pi^{JFB}}{\pi_H^D - \pi_H^N}$. Whenever this condition is satisfied, a RC will be possible no matter how impatient the supplier is.

Thus, because $\underline{\delta_M} \in (0,1)$ and δ_M is continuous on $(0,1)$ for all $\delta_H \in (\underline{\delta_H}, \overline{\delta_H})$ there will exist a range of δ_M, namely $(0, \underline{\delta_M})$, where Nash will be played and a range $[\underline{\delta_M}, 1)$ where a RC is implementable. □

Now suppose that the initially matched supplier has discount factor $\widetilde{\delta}_M$. Then in order that it is profitable for the firm to re-match, the expected gains $E[\pi_H^{RM}]$ must be larger than the continued payoffs from the current relationship, i.e.

$$E[\pi_H^{RM}] > \frac{\pi_H^i(\widetilde{\delta}_M)}{1-\delta_H}, \text{ where } i = \begin{cases} RC \text{ if } \widetilde{\delta}_M > \underline{\delta_M} \\ N \text{ otherwise} \end{cases}. \tag{31}$$

Note, that for every match that the firm encounters in the process of re-matching equation (31) must be re-evaluated with the new realization of δ_M. Note, that the firm's per period payoff π_H^N with any impatient supplier, i.e. with $\delta_M < \underline{\delta_M}$, is independent of δ. We will therefore drop the current realization index for the impatient supplier case in the following. This fact bears the additional conclusion that whenever the firm engages in re-matching an impatient supplier it will only stop the re-matching process whenever it finds a patient supplier that allows for an

RC. The expected payoffs from re-matching the current supplier can be expressed as follows:

$$E[\pi_H^{RM}] = \pi_H^N - F + \frac{\delta_H}{1-\delta_H}\left[(1-G(\underline{\delta_M}))E[\pi_H^{RC} \mid \delta_M \geq \underline{\delta_M}] + G(\underline{\delta_M})\pi_H^N\right]$$

With this expression at hand, we can solve both cases of (31) for F and obtain:

$$\begin{aligned} F &\leq \frac{\delta_H(1-G(\underline{\delta_M}))}{1-\delta_H}\left[E[\pi_H^{RC} \mid \delta_M \geq \underline{\delta_M}] - \pi_H^N\right] && \equiv \overline{F}^3 \text{ , if } i = N \\ F &\leq \frac{\delta_H(1-G(\underline{\delta_M}))}{1-\delta_H}\left[E[\pi_H^{RC} \mid \delta_M \geq \underline{\delta_M}] - \pi_H^N\right] - \frac{\widetilde{\pi_H^{RC}} - \pi_H^N}{1-\delta_H} && \equiv \overline{F}^4(\widetilde{\delta}_M) \text{ , if } i = RC \end{aligned}$$

The expressions state that whenever $F < \overline{F}^k(\widetilde{\delta}_M)$, $k \in \{3,4\}$, is fulfilled the firm will re-match its current supplier and not do so otherwise. Observe, that while the search cost threshold $\overline{F}^3$ is constant for all $\widetilde{\delta}_M < \underline{\delta}_M$, $\overline{F}^4$ is monotonically decreasing in $\widetilde{\delta}_M$ for all patient agents. Note also, that for all patience levels $\overline{F}^4(\widetilde{\delta}_M) < \overline{F}^3$ holds.

As before, in order to characterize the re-matching behavior of firms we need to rule out off-equilibrium "cheat-and-run" (CAR) behavior. For this, suppose that we are matched with some $\widetilde{\delta}_M > \underline{\delta_M}$ and (31) does not hold, i.e. we do not re-match the current supplier. Formally:

$$\frac{\pi_H^{RC}(\widetilde{\delta}_M)}{1-\delta_H} \geq E[\pi_H^{RM}] \tag{32}$$

Simultaneously, for CAR not to occur we must have:

$$\frac{\pi_H^{RC}(\widetilde{\delta}_M)}{1-\delta_H} \geq E[\pi_H^{CAR}], \tag{33}$$

where $E[\pi_H^{CAR}]$ is the solution to:

$$\begin{aligned} V_0 &= \pi_H^D - F + \delta_H V_1 \\ V_1 &= (1-G(\underline{\delta_M}))\pi_H^D + G(\underline{\delta_M})\pi_H^N - F + \delta_H V_1 \quad \Leftrightarrow V_1 = \frac{(1-G(\underline{\delta_M}))\pi_H^D + G(\underline{\delta_M})\pi_H^N - F}{1-\delta_H} \end{aligned}$$

Inspection of (32) and (33) reveals that for CAR not to occur it is sufficient that $E[\pi_H^{RM}] > E[\pi_H^{CAR}]$ holds, which can be reduced to the condition of a lower bound $\widetilde{F}$ on F:

$$F > \pi_H^N + (1-G(\underline{\delta_M}))(\pi_H^D - E[\pi_H^{RC} \mid \delta_M \geq \underline{\delta_M}]) \equiv \widetilde{F}$$

With this bound at hand we can formulate the following proposition.

Proposition 9. Suppose the headquarter is currently matched to a supplier with patience level $\widetilde{\delta}_M \in (0,1)$, for which $\widetilde{\delta} < \underline{\delta}$ ($\widetilde{\delta} \geq \underline{\delta}$) holds.

a) Then, whenever she faces search costs $F \geq \overline{F}^3(\widetilde{\delta}_M)$ *(*$F \geq \overline{F}^4(\widetilde{\delta}_M)$*) she will engage in a LTC with this supplier in the form of repeated Nash bargainings (a relational contract).*

b) On the other hand, suppose that $\widetilde{F} < F < \overline{F}^3(\widetilde{\delta}_M)$ *(*$\widetilde{F} < F < \overline{F}^4(\widetilde{\delta}_M)$*). Then the firm will play Nash with the current supplier and re-match him at the end of the initial period. The firm will continue re-matching until, in period t, she finds a supplier for which* $\delta_M^{t+1} \geq \underline{\delta}$ *and* $F \geq \overline{F}^4(\delta_M^{t+1})$ *hold. With this supplier, the firm will start a long-termed RC.*

This immediately leads to the following Corollary:

Corollary 2. *Whenever the firm decides to start re-matching, any future LTC will involve a relational contract. However, if she engages in a LTC right away, repeated Nash play cannot be ruled out.*

D "Carrot-and-stick" punishment

This Appendix documents that the main result from Proposition 1, namely the independence of the critical discount factor of A, c_m, and c_h, also holds for the case of optimal penal codes. For this section we assume that $\delta_H = \delta_M = \delta$. Consider the following carrot-and-stick strategy profile along the lines of Abreu (1988): The game starts in the cooperative state. After a deviation of player i in period t, players min-max each other for $T-1$ periods, where $T \geq 2$, beginning in period $t+1$. In period $t+T$ the game switches back to the cooperative state if both players carried out the punishment, otherwise the punishment is repeated.

The following expressions characterize the incentive compatibility constraints of the strategy profile, where (IC-on) and (IC-off) denote the constraints of player $i = \{H, M\}$ on the equilibrium path and, respectively, in the first post-deviation period. Note that the min-max-payoffs are attained at $h = m = 0$, which results in punishment payoffs equal to zero for both players.

$$\frac{\pi_i^{RC}}{1-\delta} \geq \pi_i^D + 0 \cdot \sum_{j=1}^{T-1} \delta^j + \frac{\delta^T}{1-\delta}\pi_i^{RC} \tag{IC$_i$-on}$$

$$0 \cdot \sum_{j=0}^{T-2} \delta^j + \frac{\delta^{T-1}}{1-\delta}\pi_i^{RC} \geq 0 \cdot \sum_{j=0}^{T-1} \delta^j + \frac{\delta^T}{1-\delta}\pi_i^{RC} \tag{IC$_i$-off}$$

First, consider (IC$_i$-off) and note that any deviation of player i yields at most zero when restarting the punishment. Simplifying gives $\pi_i^{RC} \geq \delta\pi_i^{RC}$, which is trivially satisfied for any discount factor. Now, consider (IC$_i$-on). Simplifying gives:

$$(1-\delta^T)\pi_i^{RC} \geq (1-\delta)\pi_i^D$$

We define the critical discount factor under the carrot-and-stick strategy as $\hat{\delta}$. Analogous to the discussion of Nash reversion in the main text, at $\delta = \hat{\delta}$, both (IC_H-on) and (IC_M-on) hold with equality. Plugging in π_i^{RC}, solving the resulting expressions for the bonus payment $B^*(\hat{\delta})$, and equalizing gives the following implicit expression for $\hat{\delta}$:

$$\hat{\delta}^T\pi^{JFB} - \hat{\delta}(\pi_H^D + \pi_M^D) + (\pi_H^D + \pi_M^D - \pi^{JFB}) = 0 \tag{34}$$

From (34) it is evident that $\hat{\delta}$ is decreasing in T. That is, the longer the punishment phase, the more stable becomes the cooperative RC. Even more importantly, since π^{JFB} and π_i^D are homogeneous of a common degree b, it also follows that the critical discount factor $\hat{\delta}$ is independent of c_m, as well as of A and c_h. The cost-orthogonality of the critical discount factor therefore also holds with this alternative penal code.

Imperfect monitoring and demand uncertainty. We now study the robustness of the cost-orthogonality result from Proposition 1 when the headquarter can only imperfectly monitor the quality of the supplier's input and demand is uncertain in a similar way as in the seminal model by Green and Porter (1984). Specifically, suppose demand realizations are stochastic and i.i.d. over periods, and in each period demand is either in a low state ($A = 0$) with probability θ, or in a high state ($A = 1$) with probability $1 - \theta$. The firm and the supplier do not know the state of demand when they make their input investments, nor can they infer it at any later point. Second, suppose the supplier M_0 has the option to supply any quantity of the input m either with a high quality ($I = 1$) or with a low quality ($I = 0$). With $I = 1$, unit costs are c_m^0 as before but the low-quality input can be supplied at zero costs for the supplier. Input quality cannot be inferred by the headquarter at any time, hence, the firm cannot disentangle if zero revenue in the last step of a stage game is due to a low state of demand or to low quality of the input. To study this variation of our model, we adjust the *stage game* as follows:

1'. **Proposal stage** (cheap talk): H can make M a non-binding and non-contractible proposal specifying investment levels (h,m) and an ex-post bonus payment B to M.

2'. **Participation decision stage**: The supplier M decides upon his participation in the relationship with H according to his outside option ω_M.

3'. **Investment stage**: The headquarter H and the supplier M simultaneously choose their non-contractible input investments (h,m). Additionally, M decides on the quality $I \in \{0, 1\}$ of his input. With $I = 0$, the input m is fully incompatible for the production of the final output and M incurs zero costs of input provision. With $I = 1$, the supplier incurs unit costs c_m as before and input m is usable for production.

4'. **Information stage**: H and M learn the investment level (quantity) of their production partner, but H cannot observe the quality I of the input.

5'. **Bargaining stage**: If a relational contract was proposed, H can decide to pay the bonus B to M. The bonus payment is made immediately (liquidity constraints are ruled out). Otherwise, the surplus is split according to an asymmetric Nash bargaining conditional on the revenue realized in 6', where $\beta \in (0, 1)$ is H's and $(1 - \beta)$ is, respectively, M's bargaining power.

6'. **Profit realization stage**:

- If $A = 0$ and/or $I = 0$ no final output can be produced and revenue is zero.
- If $A = 1$ and $I = 1$, the final output is produced and sold. The surplus is divided as specified in 5'.

We consider the following strategy profile. The game starts in the *cooperative state* in which behaviour is essentially identical to the one described in the main text. Additionally, M sets $I = 1$ in the cooperative state. Now, whenever i) an observable deviation from the RC occurs, or ii) when no final output can be sold, the game enters a *punishment phase* which lasts for T periods. In the punishment phase, H and M make zero investments and thus follow the "carrot-and-stick" penal code studied above. After the punishment phase ends, the players revert to the cooperative state.

We define by V^{+i} and V^{-i} player i's present discounted value of payoffs in a period in the cooperative state and in a period where the punishment phase has just started:

$$V^{+i} = (1 - \theta)(\pi_i^{RC} + \delta V^{+i}) + \theta\delta V^{-i}, \qquad V^{-i} = \delta^T V^{+i}$$

The solution to this system of equations is given by:

$$V^{+i} = \frac{(1-\theta)\pi_i^{RC}}{1-(1-\theta)\delta - \theta\delta^{T+1}}, \qquad V^{-i} = \frac{\delta^T(1-\theta)\pi_i^{RC}}{1-(1-\theta)\delta - \theta\delta^{T+1}}$$

In order to rationalize the relational contract, we need to set up the incentive constraints for H and M. By optimally deviating, H can attain V^{dH} and M can achieve V^{dM}, respectively. Notice that the supplier would never choose the verifiable deviation (supplying a wrong quantity $m \neq m^*$), but if he wants to deviate, he would always prefer to supply the correct quantity m^* in low quality ($I = 0$) as he will then receive the full bonus in the deviation period *before* revenue is realized.

$$V^{dH} = (1-\theta)\pi_H^D + \delta V^{-H}, \qquad V^{dM} = B + \delta V^{-M}$$

The IC-constraints, which generally read $V^{+i} \geq V^{di}$, $i = H, M$, can then be written as:

$$(1-\delta^{T+1})V^{+H} \geq (1-\theta)\pi_H^D \qquad (\text{IC}_H)$$
$$(1-\delta^{T+1})V^{+M} \geq B, \qquad (\text{IC}_M)$$

and plugging in V^{+H} and V^{+M} from above, we can rewrite the constraints as follows, where $\psi \equiv \frac{(1-\delta^{T+1})(1-\theta)}{1-(1-\theta)\delta-\theta\delta^{T+1}}$:

$$\psi\pi_H^{RC} - (1-\theta)\pi_H^D \geq 0$$
$$\psi\pi_M^{RC} - B \geq 0 \qquad (35)$$

Plugging π_M^{RC} into (35) the latter can be rearranged as $B \geq \frac{\psi}{1-\psi}\tau c_m m^*$, where a necessary condition for (IC_M) to hold is $\theta < \frac{\delta-\delta^{T+1}}{1+\delta-2\delta^{T+1}}$. In words, the cooperative RC cannot be sustained if the probability of low demand is too large, similar as in the collusion model by Green and Porter (1984).

Let us now define the critical discount factor above which relational contracting is incentive compatible as $\widetilde{\delta}$, and as before observe that at this point both IC-constraints bind with equality. Merging the constraints from (35), we can compute $\widetilde{\delta}$ implicitly from the following equation, which inter alia depends also on θ and T:

$$\psi\left(R^* - h^*c_h - \frac{\psi}{1-\psi}\tau c_m m^*\right) - (1-\theta)\pi_H^D = 0 \qquad (36)$$

For patience levels $\delta > \widetilde{\delta}$ the cooperative RC can be sustained in this model version, but similar as in Green and Porter (1984), we will observe periods of punishment and zero investments followed by cooperative periods with optimal (high-quality) investments. It is also possible to compute an *optimal* punishment length T^* which maximizes V^{+H} subject to (36). However, the more important observation for our purpose is that c_m can be factored out of the LHS of (36). From here, it follows immediately that $\widetilde{\delta}$ is independent of the unit cost level c_m^0 which reinforces our cost-orthogonality result from Proposition 1.