Optimal Payment Contracts in Trade Relationships*

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May 2022

Abstract

In buyer-seller trade relationships, long-term collaboration and payment contract selection are mutually dependent: While the provision of trade credit to buyers increases the stability of trade relationships, its availability varies systematically as relationships evolve. To explain this reciprocity, we model the optimal provision dynamics of trade credit when the seller's information about the buyer's type is incomplete and parties can sign contracts with limited enforceability. We investigate how self-enforcing relational contracts and formal contracts complement each other and show how their interaction determines optimal payment contract choice. We find that payment contracts can be interpreted as screening technologies and imply distinct learning opportunities for the seller about the buyer's type. When buyers are stochastically liquidity-constrained and sellers can observe their liquidity status, in line with empirical evidence the model predicts that all transitions between payment terms lead to the provision of seller trade credit in the long run.

Keywords: Payment contracts, Trade credit, Trade dynamics, Relational contracts

JEL Classification: L14, F34, D83, O16

* *Acknowledgments:* The author is grateful to Hartmut Egger and Jens Suedekum for invaluable advice and would also like to thank Pol Antràs, Costas Arkolakis, Julia Cajal Grossi, Fabrice Defever, Jonathan Eaton, Peter Egger, Matthias Fahn, Miriam Frey-Knoll, William Fuchs, Bob Gibbons, Paul Heidhues, David Hémous, Rocco Macchiavello, John Morrow, Marc Muendler, Peter Neary, Hans-Theo Normann, Jens Prüfer, Ferdinand Rauch, Philip Sauré, Tim Schmidt-Eisenlohr, Nathan Sussman, and audiences at the TRISTAN Workshop Bayreuth, Düsseldorf University, ETSG Florence, the Geneva Graduate Institute, JGU Mainz, the 6th Workshop on Relational Contracts and FIW Vienna for useful comments and suggestions. Financial support of the Joachim Herz Foundation is gratefully acknowledged.

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1 Introduction

A limited enforceability of formal contracts is a recurring challenge to the success of buyer-seller transactions. Payment contracts provide firms with a tool to shift the risks of contract non-compliance between trade partners. Relative to the date of shipment these define the timing according to which the buyer must pay the seller for traded products. On the one side, the seller can request cash in advance which eliminates the seller's risk of not receiving payment for products already delivered but exposes the buyer to the residual risk of not receiving the seller's shipment. Conversely, the seller can offer open account payment terms in which case the buyer needs to pay only after product arrival. This causes a reversion of the residual non-compliance risk between the buyer and the seller. In international trade, these risks are economically particularly relevant since the shipment of products over longer distances and across borders costs time. This implies that the choice of payment contracts goes hand-in-hand with a financing decision over the working capital involved in a transaction and, correspondingly, a decision over the provision of trade credit. Banks and insurance firms offer a comprehensive set of trade finance products that allow to reduce or eliminate the residual risks of contract non-compliance.[1] However, the share of global trade falling under their coverage is limited and a substantial share of firms rely on non-intermediated payment modes despite the ubiquitous challenge of institutional enforcement deficiencies.[2]

This self-sufficiency suggests a strong reliance of trade partners on informal, relational mechanisms to ensure contractual performance. A large literature documents that establishing long-termed, trustful trade relationships can help firms to overcome the obstructions of weak institutions and guarantee contractual performance.[3] At the same time, empirical evidence obtained in recent research points at a mutual dependence of the payment contract choice of firms and

[1] An overview on the most relevant products in international trade finance can be found in U.S. Department of Commerce (2012).

[2] This reliance has been documented for several countries. Using representative trade data from Chile, Garcia-Marin et al. (2020) show that more than 95% of export transactions from Chile are taking place on cash in advance or open account payment terms. Antràs and Foley (2015) document a very comparable usage pattern for a large U.S. poultry exporter. Cuñat (2007) documents that direct lending between buyers and sellers is economically important not only in terms of trade flows but also in terms of the overall firm liabilities. He shows that for small and medium-sized firms from the U.S. and the U.K. trade credit accounts for almost 50% of their short term debt. A review over the reasons for the high prevalence of inter-firm trade credit is available in Petersen and Rajan (1997), and our findings are complementary to them. They argue that sellers tend to have a financing cost advantage over traditional lenders due to a better ability to monitor buyers and to enforce credit repayment. In addition, trade credit gives sellers a device to price-discriminate, assure high product quality, and a tool to reduce transaction costs across repeat transactions with the same buyer.

[3] Important insights and a literature review on the role and interplay of formal and informal mechanisms in enforcing contracts can, e.g., be found in Johnson et al. (2002) and Greif (2005).

the sustained success of trade relationships. Antràs and Foley (2015) and Garcia-Marin et al. (2020) show that while payment terms powerfully predict the stability of trade relationships, their choice varies systematically with relationship age. They document that the provision of trade credit by sellers has a substantial positive impact on the stability of buyer-seller trade relationships, and their robustness to economic shocks. Moreover, while in a large share of new relationships payment is made in advance of shipment, sellers proceed to offer open account terms more frequently and provide larger amounts of trade credit to buyers as their relationships mature.

To explain these patterns, we propose a first relational contracting model of payment contract choice. Our analysis provides novel micro-foundations for the highlighted empirical patterns and shows that their validity crucially depends on the quality of information transmission between trade partners and enforcement institutions in the buyer's economy. We set up a model of repeated trade between a buyer and a seller who can sign contracts on individual transactions with limited enforceability. We investigate how relational incentives interact with the seller's choice of the trade volumes and the payment terms of transactions when information over the buyer's type is incomplete. We analyse how a payoff-maximizing seller can design stage contracts and adjust them over the course of the trade relationship to resolve contractual and informational frictions optimally.

In a first step, our study shows that payment contracts impact the stability of trade relationships by providing the seller with distinct learning opportunities over the buyer's type. Payment contracts can be interpreted as *screening technologies* and we find that the seller's information acquisition about the buyer's type is faster under cash in advance terms compared to open account terms. While under the former it is optimal for the seller to propose a stage contract that immediately separates buyers in new trade relationships, under open account terms the optimal contract pools buyer types and as a consequence information acquisition is more gradual. To understand this outcome, note first that the buyer's type relates to her discount factor and either she is fully myopic or patient. The type is fixed and the buyer's private information. Second, we assume that time elapses between the seller's investment in production and the buyer's revenue realization from product distribution to final consumers, making the buyer's type decisive for contract compliance.

The separating nature of cash in advance contracts implies a lower stability of trade relationships as these are only accepted by patient buyers. In established relationships, cash in advance terms also threaten stability due to their inflexibility in adjusting the size of the buyer's payment to unforeseen, temporary revenue shocks that the buyer may face when distributing the product. In contrast, under open account the payment size can be conditioned on final market outcomes which decreases the relationship's vulnerability to such shocks. At the same time, since open account terms are less efficient in the selection of patient buyers, destination market institutions matter for the enforcement of buyer payment. Our model predicts that while relationship stability increases with the quality of institutions under open account terms, they have no effect under cash in advance.

From this screening outcome it follows that the seller's choice between pre- and post-shipment payment terms takes place in an *inter-temporal trade-off between relationship stability and stage payoff growth*. While the strong screening efficiency of cash in advance terms has a destabilizing effect on relationships, at the same time the implied learning advantage boosts the profitability of subsequent transactions under any payment type. We find that whenever trade partners are patient enough, this trade-off is sufficient to provide unique predictions on how the seller can choose payment terms optimally over the entire course of a trade relationship. When the seller finds it optimal to transition between payment terms over time this leads to the usage of open account terms and thereby to an increasing provision of trade credit as relationships become more established. In this context, the seller initially exploits the buyer-separating nature of the cash in advance terms and by subsequently switching to open account he eliminates the risk of relationship breakdown due to buyer liquidity shocks in future transactions.

Decisive for the optimal usage pattern of payment terms is the seller's assessment of the buyer type distribution, as well as the amount of information available about the buyer's revenue situation. For both – new and established relationships – the model predicts that the seller will more likely extend trade credit to the buyer the smaller his belief of getting matched to a patient buyer in future relationships. While our transition predictions are confirmed by the external evidence summarized above, we also show that the documented patterns can only be rationalized when the seller is able to verify the buyer's revenue realizations from the distribution of products to final consumers. When this is not possible, the model predicts that requesting cash in advance from buyers is strictly preferable for sellers in established relationships. Our findings suggest that information transmission between trade partners plays a key role in explaining the financing patterns used in inter-firm trade.

In an extension of our model, we incorporate the possibility for the seller to obtain trade credit insurance from a competitive insurance market. When it comes to international trade, an important share of transactions with payment intermediation are backed by export credit insurances (cf. Van der Veer, 2015). In our model, the insurance takes over the risk of non-repayment of the trade credit and generates value for the seller through the insurer's expertise in the screening of buyers. We show that the unique identification of the optimal payment terms remains possible when insurance is available. When revenue shocks are verifiable for the seller, the model continues to predict that the provision of seller trade credit increases over the course of relationships which is consistent with the empirical findings of Antràs and Foley (2015).

Our analysis builds on several strands of literature where the first studies the financing terms of inter-firm trade. It extends the interpretation of trade credit by Smith (1987) who first acknowledged its role as a screening device for sellers to elicit information about buyer characteristics. More generally, the paper is related to a literature that sees credit rationing as a way to screen borrowers in markets with incomplete information (cf. Stiglitz and Weiss, 1981). Our model gives conditions under which, in equilibrium, trade credit is rationed either temporarily or permanently where in the former case this is due to screening considerations and in the latter case because financing trade is costly for the seller. While we focus on the self-financing of trade through the buyer and the seller, a complementary line of work investigates the rationales of firms to use trade credit instead of credit provided by external financial institutions.[4] Moreover, the article is connected to a literature on payment guarantees in international trade finance through our analysis of trade credit insurance. A concise summary of the most relevant work from this field was recently provided by Foley and Manova (2015).

Most closely related to our work is a small set of papers that studies the provision of trade credit in settings with repeated buyer-seller interaction. Their results are complementary to ours. The setup of our model features similarities to that of Antràs and Foley (2015) who investigate the impact of a financial crisis in a dynamic model of payment contract choice. While they also study transitions between payment terms over time their model does not incorporate that the information acquisition process of sellers differs fundamentally between pre- and post-shipment terms, inducing structural differences in the optimal growth patterns of transaction volumes and per-period payoffs. Garcia-Marin et al. (2020) derive conditions under which the provision of trade credit increases in attractiveness to sellers as their relationships with buyers mature. While

[4] For example, Burkart and Ellingsen (2004) derive conditions under which trade and bank credit interact either as complements or substitutes with each other. Demir and Javorcik (2018) interpret trade credit provision as a margin of firm adjustment to competitive pressures arising from globalization. Engemann et al. (2014) understand trade credit as a quality signalling device that facilitates obtaining complementary bank credits.

in their model this prediction originates from a financing advantage for sellers under trade credit terms, it originates from an improved payment flexibility for buyers in our setting. Fuchs et al. (2022) conduct a field experiment in Uganda to show that restricted access to liquidity is a key impediment to the business of buyers in developing countries. Like us, they study in a model of self-enforcing relational contracts how the distribution of products in developing markets can be implemented optimally in a dynamic setting. While in their work the buyer's credit line is fixed over time, in our model the existence and size of the optimal trade credit line can vary with the age of trade relationships.[5] Our model variant with non-verifiable revenue shocks and truthtelling incentivization is inspired by Troya-Martinez (2017) who studies relational contracting between a buyer and a seller for the situation when trade credit is provided in every transaction.

Also beyond the context of our application, the paper is related to the literature on self-enforcing relational contracts (cf. Thomas and Worrall, 1994; Baker et al., 2002; Levin, 2003). Like us, Sobel (2006), MacLeod (2007), and Kvaloy and Olsen (2009) study the interaction of formal and self-enforcing contracts in repeated game models when legal contract enforcement is probabilistic. Closely related to us is Kvaloy and Olsen (2009) who investigate a situation of repeated investment in a principal-agent setting with endogenous verifiability of the contracting terms. While in their setting verifiability is endogenized through the principal's investment in contract quality in our model the relevance of verifiability itself is endogenized through payment contract choice. The paper also adds to a growing literature on non-stationary relational contracts with adverse selection, in which contractual terms vary with relationship length. While in our paper learning about the buyer induces transitions between payment contract types, previous work has studied non-stationarities in different contexts.[6]

A further strand of related literature investigates the microeconomic aspects of learning and trade dynamics which, on the one side, considers applications to topics in international trade and,

[5]Beyond relationship aspects, the economic literature discusses further and complementary channels affecting the availability of trade credit to buyers. Common membership in business or ethnic networks tends to increase the willingness of sellers to provide trade credit (see Biggs et al., 2002; Fafchamps, 1997). Also, the level of competition among sellers is positively associated with the availability of trade credit to buyers (see Hyndman and Serio, 2010; Demir and Javorcik, 2018). In contrast to our work, these papers do not study the dynamic aspects of trade relationships.

[6]Chassang (2010) examines how agents with conflicting interests can develop successful cooperation when details about cooperation are not common knowledge. Halac (2012) studies optimal relational contracts when the value of a principal-agent relationship is not commonly known and, also, how information revelation affects the dynamics of the relationship. Yang (2013) investigates firm-internal wage dynamics when worker types are private information. Board and Meyer-ter-Vehn (2015) analyze labor markets in which firms motivate their workers through relational contracts and study the effects of on-the-job search on employment contracts. Moreover, Defever et al. (2016) study buyer-supplier relationships in international trade in which new information can initiate a relational contract between parties.

on the other side, contains papers of a purely contract-theoretic nature. Araujo et al. (2016) study how contract enforcement and trade experience shape firm trade dynamics when information about buyers is incomplete. We share with their work the probabilistic approach to contract enforcement, and the patterns of information acquisition and trade volume growth predicted by our model resemble the outcomes of their framework in the special situation when the seller continuously employs open account terms. Rauch and Watson (2003) study a matching problem between a buyer and a seller with one-sided incomplete information. They derive conditions under which starting a relationship with small trade volumes is preferable to starting with large transaction volumes from the very beginning. This pattern features a clear analogy to our model in which starting a relationship on open account terms corresponds to starting small, and on cash in advance terms to starting large. Extending beyond the scope of our analysis, Ghosh and Ray (1996) and Watson (1999, 2002) study agents' incentives to start small when information is incomplete on both sides of the market.[7]

The remainder of the paper is organized as follows. In Section 2, we introduce the building blocks of our analysis and, in Section 3, we study supply relationships under cash in advance and open account payment terms when switches between payment terms are ruled out. Section 4 introduces this possibility and we investigate the seller's optimal usage of payment terms over the course of trade relationships. In Section 5, we extend our model and incorporate the availability of trade credit insurance to the seller. Section 6 translates our most important model outcomes into empirically testable predictions. The last section concludes with a summary of our findings.

2 The model

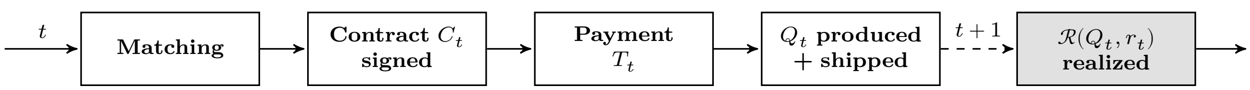
The model considers the problem of a seller ("he") who markets a product through a buyer ("she") to final consumers. There exists a continuum of potential buyers with the ability to distribute the seller's product. The seller is a monopolist for the offered product and has constant marginal production costs $c > 0$. Selling $Q_t \geq 0$ units of the product to the final consumers in period $t+1$ generates revenue $\mathcal{R}(Q_t, r_t) = r_t R(Q_t)$ to the buyer, where $R(Q_t) = Q_t^{1-\alpha}/(1-\alpha)$. The revenue function is increasing and concave in the trade volume Q_t, where $\alpha \in (0, 1)$ determines the shape of the revenue function.[8] Moreover, the revenue generated from the sales of Q_t is stochastic

[7]Beyond the case of buyer-seller transactions, relationship building has also been analyzed in the context of different applications. E.g., see Kranton (1996) and Halac (2014).

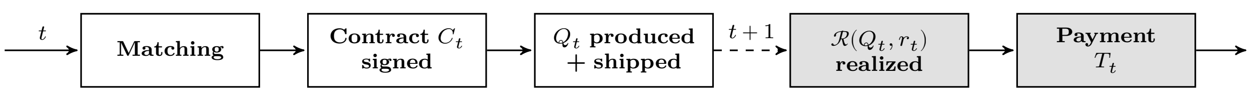
[8]Whether the concave shape of the revenue function stems from technology, preferences or market structure is not important for the analysis below.

and depends on the realization of the revenue shifter $r_t \in \{0, 1\}$. We assume that with an i.i.d. probability of $\gamma \in (0, 1)$ the revenue shifter takes value $r_t = 1$ which results in a revenue of size $R(Q_t)$. Otherwise, no revenue can be obtained from sales to final consumers.[9] The realizations of the revenue shifter are public information to both, the buyer and the seller.[10]

We model the buyer-seller relationship as a repeated game, where in every period $t = 0, 1, 2, ...$ a transaction is performed. The seller can engage in only one partnership at the same time. In every period, the seller first decides either to continue the relationship with his current buyer or to re-match and start a new partnership. He then proposes a spot contract $C_t = \{Q_t, T_t, F_t\}$ to the buyer specifying a trade volume $Q_t \geq 0$, a transfer payment T_t from the buyer to the seller, and a payment contract, $F_t \in \mathcal{F} = \{\mathrm{A}, \Omega\}$, that determines the point in time at which the transfer T_t is made. Depending on the payment contract, the seller receives the transfer either before he produces and ships the goods (cash in advance terms, $F_t = \mathrm{A}$) or after the buyer has sold them (open account terms, $F_t = \Omega$). The contract C_t therefore determines the timing of the stage game which we summarize graphically in Figure 1.



(a) *Transaction timing under cash in advance terms ($F_t = \mathrm{A}$)*



(b) *Transaction timing under open account terms ($F_t = \Omega$)*

Figure 1: *The spot contract C_t determines the timing of the stage game.*

The timing of the transfer is payoff-relevant because shipment is time-consuming and players discount payoffs over time. Goods that are produced and shipped by the seller in period t can be sold to consumers only in the subsequent period $t + 1$. The corresponding discount factor of the seller is denoted by $\delta_S \in (0, 1)$. The buyer comes in one of two possible fixed types, $j \in \{M, B\}$. Either she is fully myopic, $j = M$, with discount factor $\delta_M = 0$ and associates positive value only to payoffs of the current period. Alternatively, the buyer is patient, $j = B$, with discount

[9] Revenue non-generation can result from a variety of reasons such as unfavorable changes in consumer demand, or the destruction of the product before the sale to consumers.
Assuming full revenue destruction as the consequence of a negative shock allows us to carve out the main contributions of our analysis most concisely. In Appendix A.3, we discuss the implications of a more general shock distribution. There, we allow for arbitrary shock levels and assume that $r_t \in \{r^h, r^l\}$ with $r^h > r^l \geq 0$.

[10] In Appendix A.2, we also solve a model variant in which the realization of r_t is private information to the buyer.

factor $\delta_B \in (0,1)$. Her type is the buyer's private information. The assumptions imply that by choosing open account terms the seller extends *trade credit* to the buyer while this is not the case under cash in advance terms. Whenever the seller decides to match with a new buyer he draws her type from an i.i.d. two-point distribution, where with probability $\hat{\theta} \in (0,1)$ the buyer is myopic, and patient otherwise. We denote the seller's belief that the buyer is myopic in period t by θ_t and assume that the seller holds the belief $\theta_0 = \hat{\theta}$ at the beginning of the initial transaction with a new buyer.

Access to sufficient credit and liquidity are key obstacles to the success of firms in international trade (cf. Manova, 2013; Harrison and McMillan, 2003). We introduce liquidity constraints into the model by assuming that the buyer goes bankrupt and leaves the market whenever her realized payoff from a transaction is negative. We can infer from the timing of the stage game that while the seller can rule out any risk of buyer bankruptcy under open account terms by setting a revenue size-dependent transfer this is not possible under cash in advance terms where the transfer payment is made already before the revenue realization.[11]

In every period, the contract C_t can be enforced with an i.i.d. probability $\lambda \in (0,1)$. We think of λ as being positively associated with the quality of contract enforcement institutions in the destination market, and to be public information for all market participants. In our application, for the buyer it corresponds to the probability of not being able to deviate from making the prescribed transfer T_t and for the seller to the probability of being forced to produce and ship as agreed upon. By using this probabilistic approach of contract enforcement we follow an established literature that studies trade relationships in the presence of heterogeneous enforcement institutions (see Araujo and Ornelas, 2007; Araujo et al., 2016; Antràs and Foley, 2015).

In the following, we summarize the stage game of period t which is repeated ad infinitum.

Stage game timing.

1. ***Revenue realization.*** The level of the revenue shifter r_{t-1} is realized and learned by the buyer and the seller. The product shipped in the previous period generates revenue $\mathcal{R}(Q_{t-1}, r_{t-1})$ to the buyer from the sale to final consumers.

2. ***Payment (open account).*** The buyer decides whether to transfer T_{t-1} to the seller. She

[11] Conditioning the transfer on the realization of r_t is possible if either the revenue realization is observable for the seller, or, if the buyer truthfully reports r_t in case the realized value is her private information. In the main text, we focus on the public information case and relegate the discussion of the private information case to the Appendix.

finds an opportunity not to pay with probability $1 - \lambda$. Upon non-payment the match is permanently dissolved.

3. ***Matching.*** Whenever unmatched, the seller starts a new partnership. Otherwise, the seller chooses either to stick to the current buyer or to re-match with a new one.

4. ***Contracting.***
   - The seller decides whether to propose a one-period spot contract $C_t = \{Q_t, T_t, F_t\}$ to the buyer. The contract specifies a trade volume Q_t, a transfer T_t, and a payment contract F_t. Upon non-proposal, the match is permanently dissolved.
   - The buyer decides either to accept or to reject C_t. Upon rejection, the match is permanently dissolved.

5. ***Payment (cash in advance).*** The buyer decides whether to transfer T_t to the seller. She finds an opportunity not to pay with probability $1 - \lambda$. Upon non-payment the match is permanently dissolved.

6. ***Production and Shipment.*** The seller decides whether to produce and ship Q_t as specified in the contract. Upon non-shipment the match is permanently dissolved.

For the following, it will be helpful to define by $C = (C_t)_{t=0}^{\infty}$ the sequence of spot contracts offered by the seller over the course of the relationship. Moreover, we denote by $Q = (Q_t)_{t=0}^{\infty}$, $T = (T_t)_{t=0}^{\infty}$, and $F = (F_t)_{t=0}^{\infty}$ the corresponding sequences for trade volumes, transfer payments, and payment contracts, respectively.

3 Payment contracts in isolation

In this section, we study in isolation the two cases where the seller is restricted to choose either cash in advance or open account payment terms for all periods and rule out switches between payment terms over time. This corresponds to a situation in which the seller grants trade credit for either none or all transactions of a relationship. The possibility to vary the trade credit provision over time is introduced in Section 4 in which the seller can freely choose the payment terms in the spot contract of any transaction. This expositional approach not only allows us to highlight the different screening properties of payment contract types but also requires us to derive two repeated game equilibria that are both relevant in our study of dynamic optimality.

We consider the following *strategy profile*. In both scenarios, the seller forms a new partnership whenever unmatched. He terminates an existing partnership if and only if the buyer defaults on the contract. In any period t, the seller chooses a trade volume Q_t and a transfer profile T_t that maximize his current period expected payoffs.[12] The buyer accepts the proposed contract C_t whenever participation promises her an expected payoff at least covering her outside option. The buyer's behavior with respect to an accepted contract is determined by her type and the realization of the revenue shifter. The myopic type will deviate from any accepted contract and not pay the transfer whenever payment is not enforced. In contrast, the patient buyer is patient enough (by assumption) to never default from an accepted contract as long as she does not suffer bankruptcy. The employed equilibrium concept is that of sequential equilibrium.[13]

To simplify the exposition of our results, we normalize the outside option of the buyer to zero in the main text. In Appendix A.4, we show that our results extend to the case where the buyer's outside option is positive and, e.g., she can engage in an alternative trade relationship with a different seller.

3.1 Cash in advance terms

First, we study the case where the seller is restricted to write contracts on cash in advance terms (A-terms) only, i.e. in any trade relationship $F = (\text{A}, ...)$.[14] Under this payment sequence the seller never provides trade credit to the buyer. The participation constraint of a buyer of type $j \in \{M, B\}$ in period t is:

$$\delta_j \mathcal{R}(Q_t, r_E) - T_t \geq 0, \qquad (\text{PC}^{\text{A}}_{j,t})$$

where $r_E = \gamma$ denotes the expected value of the revenue shifter. The constraint states, that tomorrow's expected revenue $\mathcal{R}(Q_t, r_E)$ realized from the sale of today's shipment Q_t must be larger than the transfer T_t made to the seller before shipment. Because goods can be sold to final consumers only in the period following t, the revenue is multiplied by the buyer's discount factor δ_j. Observe that because $\delta_M = 0$, the myopic buyer's participation constraint, $(\text{PC}^{\text{A}}_{M,t})$, cannot be fulfilled for any $T_t > 0$. Consequently, the myopic buyer will never accept any contract

[12]Since we assume that only spot contracts are feasible and switching between payment contract types is ruled out here, the maximization of the current period expected payoffs implies that the ex-ante expected payoffs are maximized simultaneously.

[13]For adverse selection scenarios as we study them here, sequential equilibrium is the relevant notion of equilibrium, see Mailath and Samuelson (2006), pp. 158–159.

[14]In the following, in the expressions for the sequence of payment contracts F we drop the time index for notational convenience.

on A-terms and the seller offers a *separating contract* that only a patient buyer accepts. Hence, whenever a new trade relationship survives the initial transaction the seller can be certain to be matched with a patient buyer and his belief jumps from $\theta_0 = \hat{\theta}$ to $\theta_1 = 0$ and remains at this level for all further transactions with the same buyer.

While a patient buyer accepts any contract on A-terms for which ($\text{PC}^{\text{A}}_{B,t}$) is satisfied, she may suffer bankruptcy if her liquidity constraint is not satisfied. For revenue level r_t this constraint is given in period t as:

$$\delta_B \mathcal{R}(Q_t, r_t) - T_t \geq 0. \tag{$\text{LC}^{\text{A}}_{B,t}$}$$

Clearly, in the situation where $r_t = 0$ the constraint is not fulfilled for any $T_t > 0$. Since setting a non-positive transfer is never optimal for the seller, he sets the transfer to $T^{\text{A}}_t = \delta_B \mathcal{R}(Q_t, \gamma)$ such that ($\text{PC}^{\text{A}}_{B,t}$) binds and extracts the maximal amount of rents from the patient buyer that ensure her participation in the contract. In this situation, the seller accepts that the buyer goes bankrupt when the low revenue state is realized. Acknowledging this transfer strategy, the seller's trade volume choice solves the following maximization problem:

$$Q^{\text{A}}_t \equiv \arg\max_{Q_t} \pi^{\text{A}}_t = T^{\text{A}}_t - cQ_t, \tag{1}$$

i.e. he sets Q_t to maximize the difference between received transfer payment and the production costs. The optimal trade volume and the corresponding stage payoffs conditional on contract acceptance are given for all transactions on A-terms as:

$$Q^{\text{A}} = \left(\frac{\gamma \delta_B}{c}\right)^{\frac{1}{\alpha}}, \qquad \overline{\pi}^{\text{A}} \equiv \pi^{\text{A}}_t = Q^{\text{A}} \frac{c\alpha}{1-\alpha}.$$

Building on the observations above, the ex-ante expected payoffs from conducting an infinite sequence of transactions on A-terms can be derived from solving the following dynamic programming problem:

$$\begin{aligned} V^{\text{A}}_0 &= (1-\theta_0)\overline{\pi}^{\text{A}} + \delta_S \left[\gamma(1-\theta_0)V^{\text{A}}_1 + (1-\gamma(1-\theta_0))V^{\text{A}}_0\right], \\ V^{\text{A}}_1 &= \overline{\pi}^{\text{A}} + \delta_S \left[\gamma V^{\text{A}}_1 + (1-\gamma)V^{\text{A}}_0\right]. \end{aligned} \tag{2}$$

Note that a trade relationship with the same patient buyer is productive and continued only if this buyer does not go bankrupt in the respective transaction, i.e. with probability γ. Otherwise, a trade relationship with a new buyer is started. Solving the programming problem for V^{A}_0 gives the seller's ex-ante expected payoffs under A-terms, Π^{A}. They are:

$$\Pi^{\mathrm{A}} = \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A}}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)}.$$

Under A-terms, the buyer has to make the transfer before the seller's production and shipment decision. Consequently, the seller may have an incentive to deviate and not produce the output, seize the transfer, and re-match to a new buyer in the next period. To avoid this deviation, the following incentive constraint of the seller has to hold:

$$-cQ^{\mathrm{A}} + \delta_S V_1^{\mathrm{A}} \geq \delta_S V_0^{\mathrm{A}}. \qquad (\mathrm{IC}_S)$$

Lemma 1 provides parameter conditions to ensure that (IC_S) holds and guarantees equilibrium existence.[15]

Lemma 1. *Suppose that $\alpha > \tilde{\alpha} \in (0,1)$. Then there exists a repeated game equilibrium that maximizes the seller's ex-ante expected payoffs under cash in advance terms, Π^{A}, for all $\delta_S \geq \tilde{\delta}_S \in (0,1)$.*

Proof See Appendix. The parameter thresholds $\tilde{\delta}_S$ and $\tilde{\alpha}$ are defined in (A.2) and (A.3), respectively.

Some remarks on Lemma 1 are in order. For an equilibrium of the repeated game to exist the stage payoffs generated from the sale of Q^{A} units of the product must be large enough, i.e. larger than the threshold level implied by $\tilde{\alpha}$ and satisfied for all $\alpha > \tilde{\alpha}$ (since $\partial\pi_t^{\mathrm{A}}/\partial\alpha > 0$). Otherwise, a deviation by the seller cannot be ruled out since the transaction's profit margin becomes negligible and the deviation ensures the seller the full transfer at zero cost. Provided that $\alpha > \tilde{\alpha}$ holds there exist repeated game equilibria rationalizing the behavior prescribed by the strategy profile if the seller's valuation of the stream of transfers from the current buyer is high enough, as implied by the minimum discount factor $\tilde{\delta}_S$. Proposition 1 summarizes our key findings on the cash in advance equilibrium.

Proposition 1. *Suppose that payment is only possible on* A*-terms and Lemma 1 holds. Then the seller proposes a separating contract C_t that only patient buyers accept. In every period, the seller produces and ships the payoff-maximizing trade volume Q^{A}. The expected stage payoffs increase from $(1-\theta_0)\overline{\pi}^{\mathrm{A}}$ to $\overline{\pi}^{\mathrm{A}}$ after the first transaction and stay at this level for the remainder of the trade relationship. The seller's ex-ante expected payoffs are Π^{A}.*

[15] To improve readability, the explicit statement and the derivations of all parameter thresholds of the paper are omitted in the main text and can be found in the Appendix.

Proof Analysis in the text.

There are several points noteworthy about this equilibrium. First, profit maximization under cash in advance terms necessarily separates buyer types as these are very demanding for the buyer. This is demonstrated by the fact that A-terms exclude myopic buyers from cooperation altogether. For the seller, cash in advance terms have the advantage of excluding any risk of non-payment and imply that the time-invariant trade volume Q^{A} is optimal beginning with the first transaction. Moreover, all information about the buyer's type is acquired immediately with the acceptance or rejection of the initial contract C_0.[16] The stability of the trade relationship with a patient buyer depends depends exclusively on the realizations of the revenue level and is maintained as long as revenue realizations are high (i.e., $r_t = 1$).

Note also, that optimal contract design under A-terms does not depend on whether the revenue shock is realized publicly or privately to the buyer. The reason is that under A-terms the buyer's contract acceptance as well as her transfer payment decision take place before the revenue shifter is realized. This implies a contrast to the situation under Ω-terms which we study in the following section.

3.2 Open account terms

Let us now turn to the case where the seller is restricted to write contracts on open account terms (Ω-terms) only, i.e. in any trade relationship $F = (\Omega, ...)$. This case implies that trade credit is offered to the buyer in all transactions.

In contrast to the case of A-terms discussed above, under Ω-terms the buyer can make the transfer specific to the size of the realized revenue since payment is conducted subsequently. We denote by $T_t^{\Omega,h}$ and $T_t^{\Omega,l}$ the transfer that the contract of period t assigns to a high ($r_t = 1$) or, respectively, low ($r_t = 0$) realization of revenue and denote by $ET_t^{\Omega} = \gamma T_t^{\Omega,h} + (1-\gamma)T_t^{\Omega,l}$ the expected transfer payment.[17] Based on the strategy profile we can write the participation

[16]Note that the separation outcome under A-terms does not hinge on the assumption of a fully myopic buyer. Inspection of the buyer participation constraints shows that for any $\delta_M \in (0, 1)$, with $\delta_M < \delta_B$, a payoff-maximizing contract can be written that only the more patient type accepts.

[17]Alternatively, the seller can also offer a "flat" contract to the buyer specifying a transfer level that is independent of the revenue realization. We show in the Appendix, that while this approach is payoff-maximizing when revenue realizations are private information to the buyer it is payoff-dominated in the public information case.

constraints of the two buyer types for a period t contract as:

$$\gamma R(Q_t) - ET_t^{\Omega} \geq 0, \tag{PC$^{\Omega}_{B,t}$}$$

$$\gamma R(Q_t) - \lambda ET_t^{\Omega} \geq 0, \tag{PC$^{\Omega}_{M,t}$}$$

where (PC$^{\Omega}_{B,t}$) is the participation constraint of the patient buyer and (PC$^{\Omega}_{M,t}$) that of the myopic buyer, respectively. A comparison of the constraints reveals that under Ω-terms it is impossible to construct a separating contract that would guarantee to select only patient buyers. The reasons are twofold. First, myopic buyers anticipate to transfer a share of the generated revenue only if the seller can enforce the contract. This happens with probability λ and makes their PC more lenient compared to that of the patient type. Second, discounting does not affect the buyer's participation decision since both, revenue realization and payment for a period t contract happen in period $t+1$. Consequently, any feasible transaction on open account terms involves a *pooling contract*.

Suppose now that buyers behave as prescribed by the strategy profile and consider the seller's belief on the buyer's type. If the risk of buyer bankruptcy is ruled out (which the seller does by setting the state-contingent transfers accordingly, see below) then patient buyers will never deviate and myopic types do so whenever possible (i.e. they do not make the transfer when contracts can not be enforced). Hence, if no deviation occurs up to the tth transaction with the same buyer, the seller's belief of facing a myopic type in period t is given by Bayes' rule as:

$$\theta_t^{\Omega} = \frac{\hat{\theta}\lambda^t}{1-\hat{\theta}(1-\lambda^t)}. \tag{3}$$

Using equation (3), the payment probability in period t of a relationship can be written as $\Lambda(t, \hat{\theta}, \lambda) = 1 - \theta_t^{\Omega}(1-\lambda) = [1-\hat{\theta}(1-\lambda^{t+1})]/[1-\hat{\theta}(1-\lambda^t)] \equiv \Lambda_t$. Note that $\lim_{t\to\infty} \theta_t^{\Omega} = 0$ and $\lim_{t\to\infty} \Lambda_t = 1$, i.e. as the relationship with a buyer continues the seller's belief of being matched with a myopic type converges to zero while the associated payment probability converges to one. In the following, we will refer to this limiting situation in which the seller is sure to be matched with a patient buyer as the *full information limit*.[18]

Equipped with this notion of belief formation and updating, the seller's expected stage payoff

[18]Our concept of probabilistic contract enforcement (defined in section 2) assumes implicitly that the seller is not able to distinguish whether payment follows from the intrinsic motives of the (patient) buyer, or whether institutions enforce the (myopic) buyer's compliance with the contract. In Appendix A.5, we show that our qualitative findings on payment contracts (which we summarize in section 3.3) remain valid when the seller can make this distinction and is able to observe when courts are used to enforce payment. In doing so we are able to account for the observations of Macaulay (1963) who documents that business relationships often die once courts are used to enforce contract terms.

function takes the following form:

$$\pi_t^{\Omega} = \delta_S \Lambda_t \left[\gamma T_t^{\Omega,h} + (1-\gamma) T_t^{\Omega,l} \right] - cQ_t. \tag{4}$$

While the seller has to bear the costs of production cQ_t already in period t, he receives the expected transfer $\Lambda_t ET_t^{\Omega}$ only in the following period which is therefore discounted by δ_S.

Under open account terms, when deciding on the revenue-contingent transfers $T_t^{\Omega,h}$ and $T_t^{\Omega,l}$ the seller faces two challenges. First, he must ensure that the patient buyer's liquidity constraint is fulfilled for both possible realizations of the revenue level. Formally, the following constraints must hold:

$$\mathcal{R}(Q_t, 0) - T_t^{\Omega,l} \geq 0, \tag{LC$_{B,t}^{\Omega,l}$}$$

$$\mathcal{R}(Q_t, 1) - T_t^{\Omega,h} \geq 0. \tag{LC$_{B,t}^{\Omega,h}$}$$

Since the buyer can foresee that she will go bankrupt upon payment of the transfer when the respective liquidity constraint does not hold, she will instead keep the revenue for herself in this situation and accept that the relationship is discontinued (as prescribed by the strategy profile). This also implies that it is optimal for the seller to offer a contract with revenue-contingent transfers.

Second, it is not enough to merely account for the participation and liquidity constraints to guarantee that the patient buyer does not deviate from the contract. In addition, she must be incentivized by the expected payoffs of future transactions to pay the transfer instead of seizing the period's entire revenue and accept being re-matched. Formally, this gives rise to the following revenue state-contingent incentive constraints:[19]

$$-T_t^{\Omega,l} + \frac{\delta_B}{1-\delta_B}[\gamma R(Q_t) - ET_t^{\Omega}] \geq 0, \tag{IC$_{B,t}^{\Omega,l}$}$$

$$-T_t^{\Omega,h} + \frac{\delta_B}{1-\delta_B}[\gamma R(Q_t) - ET_t^{\Omega}] \geq 0. \tag{IC$_{B,t}^{\Omega,h}$}$$

The following Lemma 2 derives conditions that ensure the buyer to behave according to the strategy profile, while maximizing the seller's stage game payoffs.

Lemma 2. *Under Ω-terms, the seller sets transfers $T_t^{\Omega,l} = 0$ and $T_t^{\Omega,h} = \delta_B \gamma / (1 - \delta_B(1 - \gamma)) R(Q_t)$. Thereby, he rules out the buyer bankruptcy risk, makes the patient buyer indifferent*

[19] We assume that buyers are unaware of the seller's belief formation process and expect the terms of future contracts C_k, with $k > t$, to be identical to those of the contract signed in period t. This assumption implies that the buyer conditions her behavior on the same information set under both, A- and Ω-terms, which generates valuable tractability for the analysis in Section 4.

between paying and not paying the agreed upon transfer in any revenue state and maximizes his own payoffs.

Proof See Appendix.

Acknowledging the results of Lemma 2, the seller chooses the trade volume in period t by maximizing the following variant of (4):

$$Q_t^\Omega \equiv \arg\max_{Q_t} \delta_S \Lambda_t \mathcal{T} R(Q_t) - cQ_t, \qquad \text{where} \quad \mathcal{T} = \frac{\delta_B \gamma^2}{1-\delta_B(1-\gamma)}.$$

The optimal trade volume Q_t^Ω and the corresponding stage game payoff π_t^Ω in the tth transaction with a buyer on open account terms can be calculated as:

$$Q_t^\Omega = \left(\frac{\delta_S \mathcal{T} \Lambda_t}{c}\right)^{\frac{1}{\alpha}}, \qquad \pi_t^\Omega = Q_t^\Omega \frac{c\alpha}{1-\alpha}.$$

We define the trade volume and stage payoffs at the full information limit as $Q^\Omega \equiv \lim_{t\to\infty} Q_t^\Omega = (\delta_S \mathcal{T}/c)^{1/\alpha}$ and $\overline{\pi}^\Omega \equiv \lim_{t\to\infty} \pi_t^\Omega = Q^\Omega c\alpha/(1-\alpha)$, respectively.[20]

The seller's ex-ante expected payoff from a trade relationship on open account terms, Π^Ω, can be obtained from solving the following dynamic programming problem for V_0^Ω:

$$\forall t \geq 0: \quad V_t^\Omega = \pi_t^\Omega + \delta_S \left(\Lambda_t V_{t+1}^\Omega + (1-\Lambda_t) V_0^\Omega\right). \tag{5}$$

In the Appendix, we derive the following solution to this problem:

$$\Pi^\Omega = \frac{1-\delta_S\lambda}{1-\delta_S\lambda-\delta_S\theta_0(1-\lambda)} \overline{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} (1-\theta_0(1-\lambda^t)).$$

We summarize our findings on the open account equilibrium in Proposition 2.

Proposition 2. *Suppose that payments are only possible on Ω-terms. Then the seller proposes a pooling contract to the buyer and updates his belief as prescribed by θ_t^Ω as the relationship proceeds. Based on this belief, the trade volume Q_t^Ω (the expected stage payoffs π_t^Ω) increase gradually with the age of the relationship and converge to the full information level Q^Ω ($\overline{\pi}^\Omega$). The ex-ante expected payoffs of the seller are Π^Ω.*

Proof Analysis in the text.

[20]For later use, note that the expected stage payoffs under belief θ_t^Ω can be rewritten as an expression that is proportional to the stage payoffs at the full information limit, i.e. $\pi_t^\Omega = \Lambda_t^{\frac{1}{\alpha}} \overline{\pi}^\Omega$.

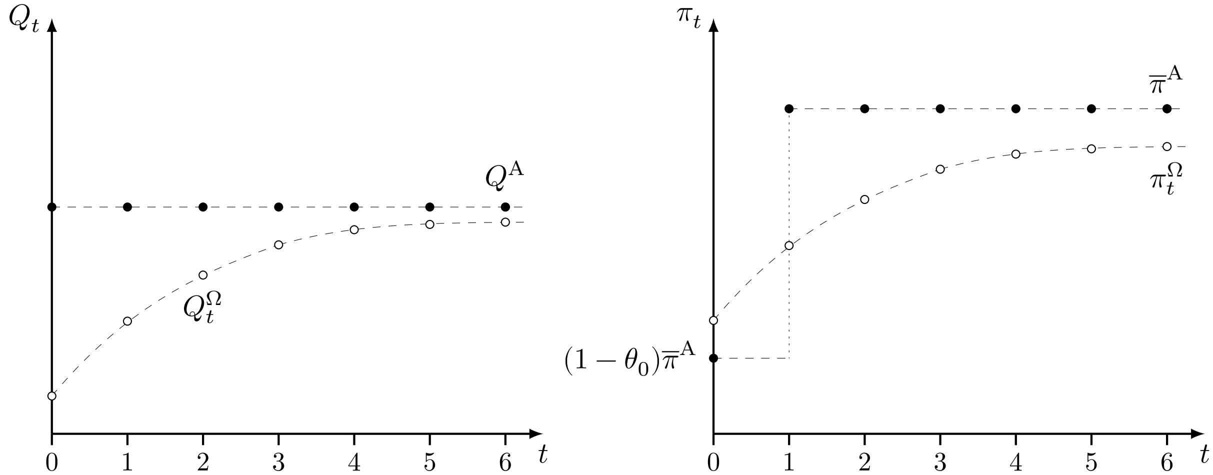
3.3 Discussion

A comparison of the results of Sections 3.1 and 3.2 reveals important differences between cash in advance and open account payment terms. On the one side, they can be summarized as features related to the *learning process* about the buyer, and to the *risks of relationship breakdown* on the other side.

First, consider the learning process about the buyer in a new relationship. Under cash in advance terms, the seller optimally offers a separating stage contract that immediately reveals the buyer's type. In contrast, immediate separation is not possible under Ω-terms where the payoff-maximizing stage contract necessarily features the pooling of buyer types. In this case, type information is acquired only gradually over time through the Bayesian updating process (see equation 3). Type separation under A-terms translates into a comparably high trade volume Q^{A} from the very first transaction while trade volumes under Ω-terms grow over time and converge to the belief-fee level Q^{Ω} as the relationship matures. This has immediate repercussions on the development of the expected stage payoffs over the course of a trade relationship. While under A-terms the expected stage payoffs jump immediately after the first successful transaction from $(1-\theta_0)\overline{\pi}^{\mathrm{A}}$ to $\overline{\pi}^{\mathrm{A}}$ and remain at this level for all following periods with the same buyer they increase at a strictly slower rate under Ω-terms as determined by the Bayesian updating process up to the level $\overline{\pi}^{\Omega}$. Figure 2 summarizes graphically the evolution of trade volumes and the seller's expected stage payoffs over the course of a trade relationship. The figure shows the payoff expectation as formed at the beginning of the contracting stage in the tth transaction with the same buyer.[21]

Second, let us compare the risks of transaction failure across payment terms. Under the considered strategy profile, transaction failure directly corresponds to the breakdown of the trade relationship with a buyer. It turns out that while under A-terms transaction failure is triggered by buyer characteristics (i.e., her type and/or liquidity status) under Ω-terms the institutional environment in which the transaction takes place is decisive. Under the latter, a transaction can be unsuccessful only if contracts cannot be enforced which induces the *non-payment* of the transfer in a match with a myopic buyer. In contrast, A-terms do not involve any payment risk for the seller since the transfer is made already before production and shipment. However, the

[21]Note that $\lim_{\delta_B\to 1}\lim_{\delta_S\to 1} Q^{\Omega} = Q^{\mathrm{A}}$ and $\lim_{\delta_B\to 1}\lim_{\delta_S\to 1}\overline{\pi}^{\Omega} = \overline{\pi}^{\mathrm{A}}$, i.e. the trade volumes and stage payoffs at the full information limit under A- and Ω-terms converge as both, the seller and the patient buyer become very patient. In any other case we have $Q^{\Omega} < Q^{\mathrm{A}}$ and $\overline{\pi}^{\Omega} < \overline{\pi}^{\mathrm{A}}$. Figure 2b depicts the situation where at $t=0$ the expected stage payoff is larger under Ω- than under A-terms which holds true if and only if $\alpha > \ln\Lambda_0/\ln(1-\theta_0) \in (0,1)$. The reverse scenario can also occur in equilibrium.



(a) *Trade volumes.* **(b)** *Expected stage payoffs.*

Figure 2: *Trade volumes and expected stage payoffs (at the contracting stage).*

latter can still result in an unsuccessful transaction in case of a match with a myopic buyer which leads to buyer *non-participation.* In addition, while low realizations of the revenue shifter cause relationship breakdown under A-terms due to buyer illiquidity, this never occurs under Ω-terms where the optimal transfer conditions on the size of the realized revenue.

Ex-ante to contracting, the probability of transaction failure in period t is given for payment contract type $i \in \mathcal{F}$ and belief θ_t as $P_t^{\mathrm{A}} = 1 - \gamma(1-\theta_t)$ and $P_t^{\Omega} = \theta_t(1-\lambda)$, respectively. Evidently, it holds that $P_t^{\Omega} < P_t^{\mathrm{A}}$ and, moreover, the seller can benefit from a smaller risk of transaction failure under Ω-terms the stronger contracting institutions are.

As a consequence, when deciding whether or not to provide trade credit to a new buyer (i.e., whether or not to offer payment on Ω-terms) the seller has to weigh the relationship stability-enhancing advantages of trade credit with the associated, comparably slow learning process about the buyer and the corresponding moderate growth of stage payoffs on the equilibrium path. In the following section, we study how the seller can manage this *trade-off between relationship stability and stage payoff growth* efficiently.[22]

[22] In Appendix A.2, we show that this trade-off remains an important determinant of payment contract choice also when revenue realizations are private information to the buyer.

4 Dynamically optimal payment contracts

4.1 Main results

We now study the seller's optimal choice of payment contracts when he can separately decide between A- and Ω-terms – and hence about the provision of trade credit – for every period of the repeated game, i.e. $F_t \in \mathcal{F}$ for all $t \geq 0$. This will give us an understanding of how the inter-temporal trade-off outlined in the previous section affects and determines the optimal choice of payment contracts in the dynamic context.

Definition The sequence F that maximizes the seller's ex-ante expected payoffs from the trade relationship is called the *dynamically optimal sequence of payment contracts* (DOSPC).

Determining the DOSPC from a direct comparison of all available sequences is impossible since this set contains infinitely many elements as a consequence of the infinite time horizon of the game. However, simple parameter refinements allow us to endogenously reduce the set of possibly optimal sequences to three elements while maintaining the presence of the inter-temporal trade-off outlined in Section 3.3.

Proposition 3. *For all parametrizations of the model satisfying the constraints* $\alpha > \underline{\alpha} \in (0,1)$ *and* $\delta_B > \underline{\delta}_B \in (0,1)$ *there exists a unique* $\underline{\delta}_S \in (0,1)$ *such that for all* $\delta_S > \underline{\delta}_S$ *we have* $F \in \{(\mathrm{A}, ...), (\Omega, ...), (\mathrm{A}, \Omega, \Omega, ...)\} \equiv \mathcal{F}^D$ *as the DOSPC.*

Proof See Appendix. The parameter thresholds $\underline{\alpha}$, $\underline{\delta}_S$, and $\underline{\delta}_B$ are defined in (A.10) and (A.11).

The parameter constraints used to derive Proposition 3 address two distinct incentive problems. The first addresses the seller's motivation to switch between payment terms over the course of a trade relationship. In our proof, we begin by analyzing optimal payment contract choice for the limiting initial beliefs, i.e. for $\theta_0 \to 0$ and $\theta_0 \to 1$. We show that in the initial transaction of a new trade relationship usage of both, A- and Ω-terms, can be optimal and hence, switches away from either payment modality over time must be considered.[23] In this context, observe that any relationship that starts on A-terms reaches the full information limit after the first

[23] When $\theta_0 \to 1$, i.e. in the situation where all buyers are myopic, choosing A-terms is never profitable as any contract on such terms will never be accepted by myopic buyers. Conversely, under Ω-terms seller profit expectations are positive, as myopic buyers accept the stage contract in which case institutions enforce the transfer with positive probability. Hence, $F = (\Omega, \ldots)$ is optimal in this case. When $\theta_0 \to 0$, all potential buyers are patient and accept contracts on A-terms. In this case, advance payment makes the seller strictly better off than open account (since, on the latter terms, the seller receives payment only one period later). Consequently, $F = (\mathrm{A}, ...)$ is optimal for the seller.

successful transaction. Consequently, given that the first transaction is conducted on A-terms, either the sequence (A, ...) or (A, Ω, Ω, ...) must be optimal. Finally, we show that whenever the trade relationship starts on Ω-terms, switches to A-terms in later periods are never optimal for the seller. Intuitively, this is the case because the informational gains under Ω-terms relative to those under A-terms are smallest in the initial transaction. Hence, whenever Ω-terms payoff-dominate in the initial transaction for the seller, they also do so in later periods. Note that a necessary requirement for any sequence other than (A, ...) to become optimal is that the seller is sufficiently patient, as payment under Ω-terms occurs only in the following period. For this, $\delta_S > \underline{\delta}_S$ is a sufficient condition.

A second set of incentive constraints is concerned with the non-shipment deviation that the seller may find optimal under A-terms. For the given set $\mathcal{F}^D$, such deviations may occur for the sequences (A, ...) and (A, Ω, Ω, ...). While Lemma 1 contains the relevant parameter constraints on α and δ_S for the former sequence, in Proposition 3 we derive additional, equivalent conditions for the latter. Ensuring product shipment for $F = (\mathrm{A}, \Omega, \Omega, ...)$ in the initial transaction additionally requires that the discount factor of the patient buyer is high enough ($\delta_B > \underline{\delta}_B$). The reason is that the expected buyer payment under Ω-terms, ET_t^{Ω} in periods $t > 0$, depends positively on her discount factor. Hence, low values of δ_B make shipment for the seller less attractive as his expected payoffs under Ω-terms in later transactions are small.

Summing up, Proposition 3 uncovers that when the trade partners are patient enough and when trade is profitable enough for the seller (as implied by $\alpha > \underline{\alpha}$) the trade-off between relationship stability and information acquisition about the buyer outlined in Section 3.3 is sufficient to reduce the set of feasible DOSPCs to $\mathcal{F}^D$. The following Corollary 1 goes one step further by showing how the seller can resolve the trade-off efficiently and demonstrates under which conditions either of the sequences is dynamically optimal. It turns out that this critically depends on the distribution of buyer types (and, correspondingly, the seller's initial belief θ_0).

Corollary 1. *(a) Under the conditions of Proposition 3 there exists a unique belief threshold $\underline{\theta}_0 \in (0, 1)$ such that the DOSPC is $F = (\mathrm{A}, ...)$ if $\theta_0 < \underline{\theta}_0$. For both sequences $F \in \{(\mathrm{A}, \Omega, \Omega, ...), (\Omega, ...)\}$ there exist parameter values $\theta_0 \in (\underline{\theta}_0, 1)$ such that either of the two is optimal. For $\theta_0 \to 1$, the DOSPC is $F = (\Omega, ...)$.*

(b) If in addition to the conditions of Proposition 3 it holds that $\alpha > \overline{\alpha} \in (0, 1)$ and $\lambda < \overline{\lambda} \in (0, 1)$, then there exists a unique $\overline{\theta}_0$ with $0 < \underline{\theta}_0 < \overline{\theta}_0 < 1$ such that the DOSPC is determined as follows:

– $F = (\mathrm{A}, ...)$ *if* $\theta_0 < \underline{\theta}_0$,

– $F = (\mathrm{A}, \Omega, \Omega, ...)$ *if* $\theta_0 \in (\underline{\theta}_0, \overline{\theta}_0)$,

– $F = (\Omega \, ...)$ *if* $\theta_0 > \overline{\theta}_0$.

Proof See Appendix. Parameter thresholds available in explicit form are defined in (A.8) and (A.13).

Figure 3 provides a graphical summary of the results in Corollary 1. It shows the seller's ex-ante expected payoffs resulting from any of the payment sequences in $\mathcal{F}^D$ as a function of the seller's initial belief that the buyer is myopic, θ_0.[24] For given $\theta_0 \in (0, 1)$, the seller will choose the payment sequence which gives him the highest ex-ante expected payoffs (as indicated by the solid line segments). We find that for both – new and established relationships that survive the initial transaction – the usage of Ω-terms and therefore the provision of seller trade credit is more likely optimal the higher belief θ_0, and correspondingly, the larger the population share of myopic buyers.[25] We elaborate on the reasons for this pattern in the following.

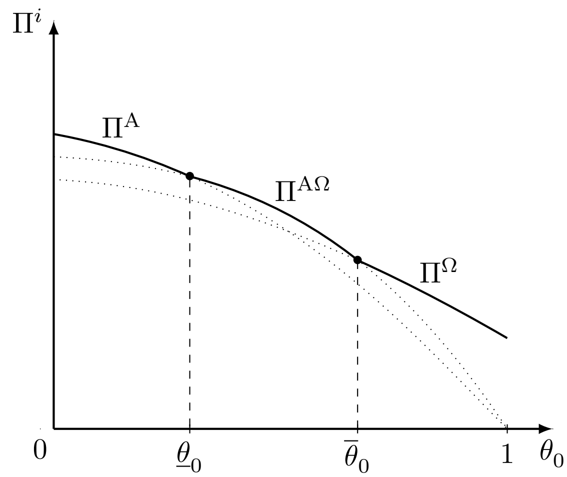


Figure 3: *Ex-ante expected payoff functions in θ_0-space.*

Consider first the situation in a newly matched buyer-seller relationship. Given the sequences in the set $\mathcal{F}^D$, exclusively the design of C_0 determines how the inter-temporal trade-off between

[24] Figure 3 shows the situation under the conditions of Corollary 1(b).

[25] To be precise, for new relationships this pattern follows only under the parameter conditions of Corollary 1(b). The upper bound on institutional quality, $\lambda < \overline{\lambda}$, ensures that Π^{Ω} is strictly concave in θ_0 which guarantees the uniqueness of $\overline{\theta}_0$. Intuitively, when the likelihood of contract enforcement is low the profitability of trade under Ω-terms is highly dependent on the distribution of buyer types which disciplines the dependence of the profit function Π^{Ω} on the seller's initial belief θ_0.

relationship stability and payoff growth is resolved optimally. Corollary 1 shows that the mitigation of relationship breakdown risks is more likely prioritized to acquiring new information about the buyer the higher the initial belief θ_0 of drawing a myopic buyer. If θ_0 is large then conducting an initial transaction on A-terms is unlikely successful since only a small share of patient buyers will accept a contract on these payment terms. This reduces the ex-ante expected payoffs associated with the sequences $(\mathrm{A}, ...)$ and $(\mathrm{A}, \Omega, \Omega, ...)$ and makes their optimality less likely.

In order to understand the rationale for varying payment terms over time it is necessary and sufficient under the restrictions of Proposition 3 to focus on the situation where A-terms are used initially. This is because the only sequence in $\mathcal{F}^D$ that contains switches between payment terms over time is $F = (\mathrm{A}, \Omega, \Omega, ...)$. While the expected stage payoffs in a non-initial transaction are larger under A-terms (i.e., $\overline{\pi}^{\mathrm{A}} > \overline{\pi}^{\Omega}$), continuing the relationship on A-terms retains carrying the risk of loosing a certainly patient buyer due to a newly arising liquidity problem. For this additional trade-off, Corollary 1 predicts that switching to Ω-terms after the initial transaction and thereby eliminating the remaining breakdown risks is preferable to obtaining a high level of stage payoffs under full information when the likelihood of finding another patient buyer is low (i.e., when $\theta_0 > \underline{\theta}_0$). In this situation, loosing the current buyer is a threat of high economic relevance to the seller and, as a consequence, he rather accepts lower stage payoffs and offers trade credit instead of risking to loose the patient buyer that he is currently matched with. Conversely, when the probability of finding a patient buyer upon relationship breakdown is high (i.e., when $\theta_0 < \underline{\theta}_0$) the seller does not find it threatful to loose his current buyer and continues business on A-terms throughout, i.e. employs $F = (\mathrm{A}, ...)$.

4.2 Discussion

Our model proposes a novel, dynamic mechanism to explain the substantial provision of trade credit by sellers and its availability to buyers engaged in international trade. It predicts that sellers are more prone to provide trade credit to their business partners the harder it is for them to find a reliable, patient buyer in the destination market and the more established the trade relationship with a particular buyer becomes. The reason is that compared to A-terms, under Ω-terms the stability of the trade relationships is not threatened by potential buyer liquidity problems which is particularly valuable when finding a reliable buyer is difficult. Stated differently, providing trade credit allows the seller to insure the trade relationship against breakdown due to unfavorable changes in buyer revenues. Whenever the seller increases the provision of trade credit to a buyer over time this originates from a learning effect about the buyer's type on the one side and causes the elimination of residual relationship breakdown risks due to potential buyer illiquidity on the other side.

In addition, the analysis shows that the different types of payment contracts can be interpreted as distinct *contract enforcement technologies*. While under Ω-terms enforcement is ensured by publicly available institutions, under A-terms it is ensured privately through the design of the contract terms which are only acceptable to reliable, patient buyers. For new trade relationships, our theory predicts that whenever the share of patient buyers is small then relying entirely on buyer selection to ensure payment (i.e. choosing A-terms for the initial transaction) is inefficient as any relationship with a myopic buyer fails immediately. In contrast, the "softer" screening under Ω-terms also allows these buyers to take up possibly productive trade relationships which has a stabilizing effect on the expected payoff stream of the seller. Overall, we show that acknowledging the screening properties of payment contracts allows to derive unambiguous recommendations on how a seller can efficiently resolve the corresponding trade-off between relationship stability and stage payoff growth.

5 Trade credit insurance

The provision of trade finance through banks and insurance firms is an important, additional driver for the growth of firms' trade volumes (cf. Amiti and Weinstein, 2011). As an example of external trade finance, we discuss the impact of the availability of trade credit insurance on dynamically optimal payment contract choice in this section.

Instead of taking the risk of buyer non-payment in an open account transaction in period t himself, the seller can rule it out by employing a trade credit insurance ($F_t = \mathrm{I}$). We assume that such an insurance is available to the seller from a perfectly competitive insurance market and that the insurance fee I_t for the transaction in period t can be separated into a fixed and a variable component which is given by:

$$I_t = m + \delta_S(1 - \Lambda_t^{\mathrm{I}})ET_t,$$

where the fixed (and time-invariant) component $m > 0$ covers setup and monitoring costs that the insurer incurs for managing the transaction. The second addend represents the variable component that depends on the size of the insured expected transfer, ET_t.[26] It is weighted by the probability of non-payment $1-\Lambda_t^{\mathrm{I}}$, where Λ_t^{I} denotes the payment probability when in the tth transaction of a trade relationship is conducted under insurance. Moreover, because potential payment default occurs only in $t+1$ the variable component is discounted. For analytical simplicity we assume the insurer's discount factor is equal to that of the seller, δ_S. Finally, because the insurer has a vital interest that the buyer does not default on the contract it will engage in buyer screening itself before granting a credit insurance.[27] We model this aspect by assuming that initially using a trade credit insurance reduces the proportion of myopic types in the population to $\widehat{\theta}^{\mathrm{I}} = \phi\widehat{\theta}$, where $\phi \in (0,1)$ is an inverse measure of the insurer's ability to screen out myopic types. Hence, the seller's belief to face a myopic buyer in the tth transaction on insurance terms is determined via Bayes' rule as $\theta_t^{\mathrm{I}} = \widehat{\theta}^{\mathrm{I}}\lambda^t/[1-\widehat{\theta}^{\mathrm{I}}(1-\lambda^t)]$, and the probability of payment in t is given as $\Lambda_t^{\mathrm{I}} = 1-\theta_t^{\mathrm{I}}(1-\lambda)$.[28]

5.1 The optimal spot contract with insurance

We employ the same strategy profile as in the baseline scenario. In addition, we assume that the seller terminates the trade relationship and matches with a new buyer whenever the buyer does make the transfer and the insurance repays instead. The participation constraints of the two buyer types under insurance are the same as in the open account scenario. Also, the incentive constraints for the patient buyer to conduct payment are the same as under open account leading the seller to request the same transfer profile from the buyer (i.e. Lemma 2 applies directly). The optimal trade volume in period t, Q_t^{I}, is hence determined by maximizing the following stage

[26] We assume that in case of buyer non-payment, the insurance reimburses the seller the factually forgone transfer, i.e. the insurer pays out $T_t^{\Omega,h}$ when $r_t = 1$, and $T_t^{\Omega,l}$ otherwise, which is consistent with perfect competition assumption for the insurance market.

[27] This assumption is endorsed by the fact that trade credit insurers such as Euler Hermes and AIG advertise their insurance services with their expertise in monitoring the reliability of transaction counterparts.
Our specification of the insurance fee follows the formalization of the letter of credit contract by Niepmann and Schmidt-Eisenlohr (2017). Its size follows from the perfect competition assumption for the insurance market which implies that the insurer makes zero profits. Since the introduction of banks as additional strategic players would render our dynamic model intractable we refrain from discussing the details of other forms of trade finance such as documentary collections and letters of credit in this paper and focus our study on the impact of the insurance on the seller's payment contract choices.

[28] In addition to having a superior ability to screen buyers, the insurance firm may be more proficient than the seller in enforcing the contract in court (e.g., due to an specialized legal department). In the model, such an ability can be introduced by assuming a higher value of the contract enforcement parameter λ under insurance. For a given belief θ_t^{I} a stronger enforcement ability of the insurer then implies a smaller insurance fee I_t in a perfectly competitive insurance market, which further increases the attractiveness for the seller to use trade credit insurance. In our analysis, we focus on the buyer selection channel.

payoff function:

$$Q_t^{\mathrm{I}} \equiv \arg\max_{Q_t} \delta_S \mathcal{T} R(Q_t) - cQ_t - I_t = \arg\max_{Q_t} \delta_S \Lambda_t^{\mathrm{I}} \mathcal{T} R(Q_t) - cQ_t - m,$$

where the second equality holds since the insured expected transfer is $ET_t = \mathcal{T} R(Q_t)$.

Observe that even though the insurance eliminates the risk of non-payment, the probability of payment Λ_t^{I} still indirectly affects the seller's maximization problem through the variable fee component. The optimal trade volume Q_t^{I} and the corresponding stage payoffs π_t^{I} are:

$$Q_t^{\mathrm{I}} = \left(\frac{\delta_S \mathcal{T} \Lambda_t^{\mathrm{I}}}{c} \right)^{\frac{1}{\alpha}}, \qquad \pi_t^{\mathrm{I}} = Q_t^{\mathrm{I}} \frac{c\alpha}{1-\alpha} - m.$$

5.2 Dynamically optimal payment contracts with insurance

In any period t, the seller can now freely choose not only between cash in advance and open account terms but can alternatively decide to use a trade credit insurance, i.e. $F_t \in \mathcal{F}^+ \equiv \{\mathrm{A}, \Omega, \mathrm{I}\}$. In the following, we study how the availability of insurance affects the set of feasible dynamically optimal payment contract sequences. In fact, under the parameter restrictions of Proposition 3 the set of possible DOSPCs is extended by one unique element in the presence of insurance terms.

Corollary 2. *Let $F_t \in \mathcal{F}^+$ for all $t \geq 0$. Under the conditions of Proposition 3, it holds that some $F \in \mathcal{F}^D \cup (\mathrm{I}, \Omega, \Omega, ...) \equiv \mathcal{F}^{D+}$ is the DOSPC. The seller's ex-ante expected payoffs for the sequence $F = (\mathrm{I}, \Omega, \Omega, ...)$ are given by:*

$$\Pi^{\mathrm{I}\Omega} = \frac{1-\delta_S\lambda}{1-\delta_S\lambda-\delta_S\theta_0^{\mathrm{I}}(1-\lambda)} \left[-m + \overline{\pi}^{\Omega} \sum_{t=0}^{\infty} \delta_S^t (\Lambda_t^{\mathrm{I}})^{\frac{1}{\alpha}} (1-\theta_0^{\mathrm{I}}(1-\lambda^t)) \right].$$

Proof See Appendix.

The proof of Corollary 2 establishes that $F = (\mathrm{I}, \Omega, \Omega, ...)$ is the only additional sequence that can become dynamically optimal. This is because, first, I-terms are payoff-dominated by Ω-terms at the full information limit and after the initial play of I-terms and, second, the informational benefit from insurer screening is largest in the initial period. The proof argues that the parameter requirements imposed in Proposition 3 are sufficient to establish that $\mathcal{F}^{D+}$ is the full set of feasible DOSPCs when insurance becomes available. Acknowledging that some $F \in \mathcal{F}^{D+}$ is optimal, the following Corollary 3 gives conditions under which insuring the initial open account transaction is payoff-maximizing for the seller.

Corollary 3. *Suppose that the parameter constraints of Corollary 1 are satisfied. Then for any level of insurer screening efficiency* $\phi \in (0,1)$ *there exist threshold levels* $\overline{m} > 0$ *and* $\hat{\theta}_0 \in (0,1)$ *such that for all* $m < \overline{m}$ *and all* $\theta_0 > \hat{\theta}_0$ *the sequence* $F = (\mathrm{I}, \Omega, \Omega, ...)$ *is the DOSPC. If* $m > \overline{m}$*, then* $F \in \mathcal{F}^D$*.*

Proof See Appendix. For parameter threshold $\overline{m}$ an explicit solution exists and can be found in (A.15).

Corollary 3 shows that no matter how efficient the insurer is in screening the population of buyers there always exists an upper bound of insurance fixed costs $\overline{m} > 0$ below which the seller finds it optimal to use $F = (\mathrm{I}, \Omega, \Omega, ...)$, provided that the marginal impact of the insurer's screening activity is high enough (i.e. the share of myopic buyers in the population is large enough). Conversely, when the fixed costs of the insurer are too large (i.e., when $m > \overline{m}$) insurance is never optimal for the seller and the set of possible DOSPCs reduces to $\mathcal{F}^D$.

6 Testable predictions

Our analysis rationalizes the empirical patterns on relationship stability and the usage of payment contract from Antràs and Foley (2015) and Garcia-Marin et al. (2020) as summarized in the introduction. At the same time, we further qualify their empirical results by showing how they rely on the institutional properties of the destination market as well as on the information exchange between trade partners. We summarize the key predictions of our model in the following.

Prediction 1. *A trade relationship (irrespective of its age) is more stable and more likely survives from one transaction to the next when payment is conducted on* Ω*-terms as compared to* A*-terms. With a better quality of contract enforcement institutions in the destination market, relationship stability increases under* Ω*-terms and is unaffected under* A*-terms.*

In our model, the higher relationship stability under Ω-terms originates from the fact that only under these terms the likelihood of buyer contract compliance benefits from institutional enforcement, and from the repayment flexibility that Ω-terms give the buyer with respect to revenue shocks (as, e.g., implied by variations in final consumer demand). Thereby, we show how shocks and relationship default systematically interact with the choice of payment terms and provide a theoretical micro-foundation to the reduced-form analysis of Antràs and Foley (2015).

Relatedly, we provide an argument why even in the absence of a large macroeconomic shock (affecting contract compliance under both, A- and Ω-terms) one should expect larger relationship discontinuation rates under A-terms.[29] We find that optimal contract design attenuates the impacts of unanticipated shocks under Ω-terms but does not do so under A-terms.

Building on these patterns, Prediction 1 also underscores that better contract enforcement institutions increase the relationship stability under Ω-terms by constraining the non-payment opportunities for buyers. In contrast, better institutions have no such effect under A-terms. The reason is that advance payment enables the seller to efficiently screen buyers for their reliability and thereby makes institutional contract enforcement redundant. This differential effect of institutional quality remains to be tested in future empirical work.

For a given seller with initial belief θ_0 the model predicts a unique DOSPC. Across individual sellers the ex-ante assessment of the buyer pool is likely heterogeneous and, e.g., does depend on the seller's experience in the destination market (cf. Araujo et al., 2016). When the initial beliefs of sellers in an industry are sufficiently dispersed and – in model terms – some sellers do have "moderate" and fixed initial beliefs with $\theta_0 \in (\underline{\theta}_0, \overline{\theta}_0)$, then the model provides the following industry-level predictions.[30]

Prediction 2. *When sellers can verify buyer revenue shocks, at the industry level the relative usage of* Ω*-terms to* A*-terms increases with the age of trade relationships. When shocks are non-verifiable, the usage of* Ω*-terms does not increase with relationship age.*

When revenue shocks are public information, in our model the main rationale to increase trade credit provision over time is to strengthen the resilience of relationships to revenue shocks.[31] While this leads to qualitatively comparable predictions on payment term transitions as in Antràs and Foley (2015) the mechanism that underlies the choice dynamics in our model is fundamentally different: In the mentioned paper transitions are generated from the differential efficiency of the banking system in the seller's and the buyer's economy. In contrast, we show that the prediction remains valid when abstracting from specific properties of the financial system and institutional differences between countries. We argue that the outlined transitions are a direct consequence

[29] Motivated by the global financial crisis in 2008, the analytical focus of the dynamic model in Antràs and Foley (2015) is on the impact of large macro-level shocks on relationship stability under different payment modes. While demand shocks in their framework reduce seller stage payoffs proportionally and cause relationship breakdown under either payment mode, our findings at the contractual level suggest that the seller's ability to condition transfer payments on shock outcomes under Ω-terms makes trade relationships systematically more stable under these terms.

[30] Prediction 2 follows from combining the theoretical results of Corollary 1 and Lemma A.2.

[31] For details on the inter-temporal choice mechanism, see the discussion in Section 4.2.

of optimal contract design when buyer revenue information is available to the seller.

The transition dynamics described above find empirical support in the transaction-level trade data analyzed in the mentioned papers which underscores the practical relevance of the public information case of our model. For the markets studied there, our model suggests that sellers are well-aware of the revenue situation of buyers as, e.g., implied by the demand fluctuations of consumers in the local buyer economy. Our model extension in Appendix A.2 points out that when sellers cannot verify the buyer's revenue situation they loose important flexibility to design an incentive-compatible repayment scheme under Ω-terms which makes providing trade credit less attractive. For this case, the model predicts that in established trade relationships sellers will never find it optimal to offer trade credit to their buyers. While the prediction on how information availability and payment term selection in trade relationships interrelate is clear cut in our model, a direct empirical test of Prediction 2 is difficult. Even though controlling for information transmission between firms may be impossible with observational trade data, an experimental setting appears to be a promising avenue to bring our informational predictions to an empirical test.

7 Conclusion

In this paper, we have used external evidence on the usage of payment terms in inter-firm trade relationships to motivate a theoretical analysis on how sellers can employ payment contracts to improve the efficiency of buyer-seller cooperation. We have developed a relational contracting model in which trade volumes and payment terms of transactions are determined endogenously, and buyer payment compliance as well as the enforcement of formal contracts are uncertain. We have shown that pre- and post-shipment payment terms inhibit structurally different learning opportunities for the seller, allowing to address and improve the efficiency of trade relationships. Deciding on whether or not to provide trade credit requires the seller to prioritize between the stability and the profitability of the exchange relationship with a buyer. We have shown that the seller can resolve this trade-off in an optimal way by assessing the distribution of buyer types, based on which new trade relationships are formed.

While it is reassuring that our model can rationalize important empirical evidence on the dynamics of firm payment contract choice (cf. Antràs and Foley, 2015), the results also suggest that the generality of the usage patterns documented in their work is limited. We have found that only if the seller can obtain reliable information on the revenues that the seller makes from final consumers can it be optimal for him to increase the provision of trade credit over time. Also beyond the topic of payment contracts, this qualifying finding points at the important role that the verifiability of information plays for the structure and evolution of trade patterns and relationships. While reliable measures on the information transmission between trade partners may be difficult to obtain from observational data, an experimental research setup in the field or the laboratory can offer a fruitful approach to bring our predictions to an empirical test.

While for the largest part of this paper the analysis has focused on the non-intermediated payment modes of cash in advance and open account, trade finance products provided by banks and insurance firms are also of practical relevance (cf. Niepmann and Schmidt-Eisenlohr, 2017). Our paper incorporates external forms of trade finance into the discussion by analyzing and identifying the impact of trade credit insurance on the dynamically optimal choice of payment contracts. While we show that the main mechanisms of our model are robust to the availability of such an insurance, a promising avenue for future research is to further explore the micro-foundations of other relevant types of external trade finance such as letters of credit and documentary collections in a dynamic contracting framework.

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A Theoretical appendix

A.1 Proofs

Proof of Lemma 1

At the Production and Shipment stage (6) of any period the seller will not deviate from the contract if and only if (IC_S) holds. The seller's incentive constraint ensures that making the effort to produce the contracted output plus the continuation payoff from the current relationship with a patient buyer results in a higher payoff than deviating by not producing and shipping the agreed quantity Q^{A}. In this latter case the current relationship breaks down and one with a new buyer is started in the following period. Plugging explicit values for V_0^{A} and V_1^{A} into (IC_S) and simplifying gives:

$$-cQ^{\mathrm{A}} + \delta_S \frac{(1-\theta_0+\gamma\theta_0(1-\delta_S))\overline{\pi}^{\mathrm{A}}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)} \geq \delta_S \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A}}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)}. \tag{A.1}$$

Observing that $cQ^{\mathrm{A}} = \overline{\pi}^{\mathrm{A}}(1-\alpha)/\alpha$ we can simplify (A.1) to:

$$\delta_S \geq \frac{1-\alpha}{\gamma\theta_0} \equiv \tilde{\delta}_S. \tag{A.2}$$

For an equilibrium to exist we need to ensure that $\tilde{\delta}_S < 1$. This is the case whenever:

$$\alpha > 1-\gamma\theta_0 \equiv \tilde{\alpha} \in (0,1) \tag{A.3}$$

holds. In this situation, the non-production deviation of the seller can be ruled if he is patient enough, i.e. when $\delta_S \geq \tilde{\delta}_S$ holds. ■

Proof of Lemma 2

In the following, we determine the transfer levels $\{T_t^{\Omega,l}, T_t^{\Omega,h}\}$ that maximize the seller's stage payoffs (and thereby also his ex-ante expected payoffs). In general, the seller chooses $\{Q_t^{\Omega}, T_t^{\Omega,l}, T_t^{\Omega,h}\}$ such that the stage payoffs in (4) are maximized, subject to ($LC_{B,t}^{\Omega,l}$), ($LC_{B,t}^{\Omega,h}$), ($IC_{B,t}^{\Omega,l}$), and ($IC_{B,t}^{\Omega,h}$). Clearly, the liquidity constraints ensure that ($PC_{B,t}^{\Omega}$) holds as well.

First, note that the seller's stage payoffs increase in both $T_t^{\Omega,l}$ and $T_t^{\Omega,h}$. We can start by requiring that ($LC_{B,t}^{\Omega,l}$) binds and set $T_t^{\Omega,l} = 0$. This simplifies ($IC_{B,t}^{\Omega,h}$) to:

$$-T_t^{\Omega,h} + \frac{\delta_B\gamma}{1-\delta_B(1-\gamma)} R(Q_t) \geq 0. \tag{A.4}$$

Observe that the maximal value of $T_t^{\Omega,h}$ for which both, (A.4) and ($LC_{B,t}^{\Omega,h}$), hold is the point where (A.4) binds with equality. Hence, the seller will set $T_t^{\Omega,h} = \frac{\delta_B\gamma}{1-\delta_B(1-\gamma)} R(Q_t)$ to extract the maximal amount of rents.

A comparison of ($IC_{B,t}^{\Omega,l}$), and ($IC_{B,t}^{\Omega,h}$) reveals that $T_t^{\Omega,h} \geq T_t^{\Omega,l}$ must hold in order for all constraints of the maximization problem to be satisfied. This is always the case. ■

Derivation of the ex-ante expected payoffs Π^Ω

This appendix complements the analysis of the main text by providing a non-recursive expression of the seller's ex-ante expected payoffs under open account terms. We proceed in two steps. First, we rewrite the period t-version of equation (5) by repeatedly substituting in the value functions of all subsequent periods. Second, we solve the resulting equation for period $t = 0$. By substituting in, we can rewrite (5) to:

$$V_t^\Omega = \overline{\pi}^\Omega \left[\Lambda_t^{\frac{1}{\alpha}} + \sum_{i=t+1}^{\infty} \delta_S^{i-t} \Lambda_i^{\frac{1}{\alpha}} \prod_{j=t}^{i-1} \Lambda_j \right] + V_0^\Omega \left[\delta_S (1 - \Lambda_t) + \sum_{i=t}^{\infty} \delta_S^{i-t+2} (1 - \Lambda_{i+1}) \prod_{j=t}^{i} \Lambda_j \right]. \quad \text{(A.5)}$$

Observing that $\prod_{j=t}^{i} \Lambda_j = (1 - \theta_0(1 - \lambda^{i+1}))/(1 - \theta_0(1 - \lambda^t))$, we can simplify (A.5) to:

$$V_t^\Omega = \frac{1}{1 - \theta_0(1 - \lambda^t)} \left[\overline{\pi}^\Omega \sum_{i=t}^{\infty} \delta_S^{i-t} \Lambda_i^{\frac{1}{\alpha}} (1 - \theta_0(1 - \lambda^i)) + \delta_S V_0^\Omega \left(\frac{\theta_0 \lambda^t (1 - \lambda)}{1 - \lambda \delta_S} \right) \right]. \quad \text{(A.6)}$$

Now suppose that $t = 0$. Solving the resulting version of (A.6) for V_0^Ω gives:

$$\Pi^\Omega = \frac{1 - \lambda \delta_S}{1 - \delta_S(\theta_0 + (1 - \theta_0)\lambda)} \overline{\pi}^\Omega \sum_{t=0}^{\infty} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} (1 - \theta_0(1 - \lambda^t)).$$

Proof of Proposition 3

For the proof, we re-express the value functions in (2) and (5) to introduce additional notation allowing us to distinguish more explicitly between the current period belief θ_t, $t \geq 0$, and the initial period belief θ_0. For payment contract type $i \in \mathcal{F}$ we denote the corresponding value function applicable in period t of the trade relationship as $V_t^i(\theta_t, \theta_0)$ in the following. We have:

$$\begin{aligned} V_t^{\mathrm{A}}(\theta_t, \theta_0) &= (1 - \theta_t)\overline{\pi}^{\mathrm{A}} + \delta_S \left[\gamma(1 - \theta_t) V_{t+1}(0, \theta_0) + (1 - \gamma(1 - \theta_t)) V_{t+1}(\theta_0, \theta_0) \right], \\ V_t^\Omega(\theta_t, \theta_0) &= \pi_t^\Omega + \delta_S \left[(1 - \theta_t(1 - \lambda)) V_{t+1}(\theta_{t+1}^\Omega, \theta_0) + \theta_t(1 - \lambda) V_{t+1}(\theta_0, \theta_0) \right], \end{aligned} \quad \text{(A.7)}$$

where $V_t(\theta_t, \theta_0) \in \{V_t^{\mathrm{A}}(\theta_t, \theta_0), V_t^\Omega(\theta_t, \theta_0)\}$. When the seller is interested in setting the DOSPC, for every belief θ_t in any period $t \geq 0$ he sets $F_t \in \mathcal{F}$ such that $V_t(\theta_t, \theta_0) = \max\{V_t^{\mathrm{A}}(\theta_t, \theta_0), V_t^\Omega(\theta_t, \theta_0)\}$. In the following steps, we derive conditions ensuring that $\mathcal{F}^D$ represents the full set of possible DOSPCs.

Step 1: For limiting initial beliefs, $\theta_0 \to 0$ and $\theta_0 \to 1$, we show that only $F_t = (\mathrm{A}, ...)$ and $F_t = (\Omega, ...)$, respectively, can be dynamically optimal.

First, consider the situation where $\theta_0 \to 1$. We get $\lim_{\theta_0 \to 1} V_t^\Omega(\theta_t, \theta_0) = \lambda^{\frac{1}{\alpha}} \overline{\pi}^\Omega/(1 - \delta_S) > \lim_{\theta_0 \to 1} V_t^{\mathrm{A}}(\theta_t, \theta_0) = 0$. Since the value function expressions are independent of θ_t, it follows that $F_t = (\Omega, ...)$ is optimal in this case. Next, consider the situation where $\theta_0 \to 0$. This gives:

$$\lim_{\theta_0 \to 0} V_t^\Omega(\theta_t, \theta_0) = \left(\frac{\delta_S \gamma}{1 - \delta_B(1 - \gamma)} \right)^{\frac{1}{\alpha}} \frac{\overline{\pi}^{\mathrm{A}}}{1 - \delta_S} < \lim_{\theta_0 \to 0} V_t^{\mathrm{A}}(\theta_t, \theta_0) = \frac{\overline{\pi}^{\mathrm{A}}}{1 - \delta_S}.$$

Again, by the independence of the expressions of θ_t, it follows that $F_t = (\mathrm{A}, ...)$ must be optimal.

Step 2: We show that if the seller is sufficiently patient the only additional payment sequence that can become dynamically optimal is $F_t = (\mathrm{A}, \Omega, \Omega, ...)$.

From Step 1, we know that both, A- and Ω-terms can be optimal in the initial period. First, let us consider the case where A-terms are chosen initially ($F_0 = \text{A}$). Then, due to the separating nature of the optimal stage contract under these terms the game reaches the full information limit in the following period given that the relationship continues. Since at this limit the game reaches an absorbing state the payment contract that is optimal in $t = 1$ is also optimal in all further periods. As a consequence, the only payment contract sequences that can become optimal when $F_0 = \text{A}$ are $(\text{A}, ...)$ and $(\text{A}, \Omega, \Omega, ...)$. At the contracting stage in $t = 1$, the seller chooses the payment terms $F_1 \in \{\text{A}, \Omega\}$ by comparing the following value functions:

$$V_1^{\text{A}}(0, \theta_0) = \frac{(1-\delta_S\theta_0)\overline{\pi}^A}{(1-\delta_S)(1-\delta_S\gamma\theta_0)} \quad \text{and} \quad V_1^{\Omega}(0, \theta_0) = \left(\frac{\delta_S\gamma}{1-\delta_B(1-\gamma)}\right)^{\frac{1}{\alpha}} \frac{\overline{\pi}^{\text{A}}}{1-\delta_S},$$

and will prefer Ω-terms over A-terms in all periods $t > 0$ if and only if:

$$V_1^{\Omega}(0, \theta_0) > V_1^{\text{A}}(0, \theta_0) \quad \Leftrightarrow \quad \theta_0 > \frac{1-\left(\frac{\delta_S\gamma}{1-\delta_B(1-\gamma)}\right)^{\frac{1}{\alpha}}}{\delta_S\left(1-\gamma\left(\frac{\delta_S\gamma}{1-\delta_B(1-\gamma)}\right)^{\frac{1}{\alpha}}\right)} \equiv \underline{\theta}_0. \tag{A.8}$$

Clearly, $\underline{\theta}_0 > 0$. Moreover, since $\partial\underline{\theta}_0/\partial\delta_S < 0$ and $\lim_{\delta_S\to 1}\underline{\theta}_0 < 1$, there exists $\delta_S' \in (0, 1)$ such that $\underline{\theta}_0 \in (0, 1)$ holds for all $\delta_S > \delta_S'$.

Second, consider the case where Ω-terms are chosen initially ($F_0 = \Omega$), in which case the seller's belief is updated according to Bayes' rule when the initial transaction is successful and $\theta_1 = \theta_1^{\Omega}$. In the following, we show that whenever it is optimal to choose Ω-terms initially, it is never optimal to switch to A-terms in a later transaction. This establishes that the DOSPC is $F = (\Omega, ...)$ in this case.

For the following arguments we first need to establish the comparative statics of the value functions with respect to the current period belief θ_t. Observe that the flow payoffs in both value functions in (A.7) are decreasing in θ_t. From this it directly follows that $\partial V_t^{\text{A}}(\theta_t, \theta_0)/\partial\theta_t < 0$ and $\partial V_t^{\Omega}(\theta_t, \theta_0)/\partial\theta_t < 0$. Moreover, the flow payoffs under A-terms and (due to the immediate buyer separation under A-terms) also $V_t^{\text{A}}(\theta_t, \theta_0)$ are linear in θ_t and, hence, $\partial^2 V_t^{\text{A}}(\theta_t, \theta_0)/\partial\theta_t^2 = 0$. In contrast, observe that:

$$\frac{\partial^2 V_t^{\Omega}(\theta_t, \theta_0)}{\partial\theta_t^2} = \frac{(1-\alpha)(1-\lambda)^2\pi_t^{\Omega}}{\alpha^2\Lambda_t^2} - 2(1-\lambda)\delta_S\frac{\partial V_{t+1}(\theta_{t+1}^{\Omega}, \theta_0)}{\partial\theta_t} + \delta_S\Lambda_t\frac{\partial^2 V_{t+1}(\theta_{t+1}^{\Omega}, \theta_0)}{\partial\theta_t^2}, \tag{A.9}$$

where $\text{sgn}(\partial V_{t+1}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_t) = \text{sgn}(\partial V_{t+1}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_{t+1}) = -1$ since $\partial\theta_{t+1}^{\Omega}/\partial\theta_t > 0$. Moreover, we conclude that $\partial^2 V_{t+1}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_t^2 \geq 0$ using a case distinction: When A-terms are chosen in $t+1$, we have $\partial^2 V_{t+1}^{\text{A}}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_t^2 = 0$. When Ω-terms are chosen in $t+1$, it follows from $\partial\theta_{t+1}^{\Omega}/\partial\theta_t > 0$ and $\partial^2\theta_{t+1}^{\Omega}/\partial\theta_t^2 > 0$ that $\text{sgn}(\partial^2 V_{t+1}^{\Omega}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_t^2) = \text{sgn}(\partial^2 V_{t+1}^{\Omega}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_{t+1}^2)$. Also note that at $\theta_t = 0$, we have:

$$\frac{\partial^2 V_t^{\Omega}(0, \theta_0)}{\partial\theta_t^2} = \frac{1}{1-\delta_S}\left[\frac{(1-\alpha)(1-\lambda)^2\overline{\pi}^{\Omega}}{\alpha^2} - 2(1-\lambda)\delta_S\frac{\partial V_{t+1}^{\Omega}(0, \theta_0)}{\partial\theta_t}\right] > 0.$$

Since the first two addends in (A.9) are positive for all $\theta_t \in [0, 1)$ it follows from the above observations that $\partial^2 V_{t+1}(\theta_{t+1}^{\Omega}, \theta_0)/\partial\theta_t^2 > 0$ in the present case. Hence, $\partial^2 V_t^{\Omega}(\theta_t, \theta_0)/\partial\theta_t^2 > 0$ holds.

From the limit properties derived in Step 1 it follows, that there exists a neighborhood of initial beliefs around the limit belief $\theta_0 \to 1$ for which $V_0^{\Omega}(\theta_0, \theta_0) > V_0^{\mathrm{A}}(\theta_0, \theta_0)$ holds, i.e. Ω-terms are chosen initially. Consider now any such level of the initial belief θ_0. In this situation, the seller evaluates the comparatively small learning gains available under Ω-terms (and as prescribed by updating rule θ_1^{Ω}) as preferable to the type-separation outcome under A-terms (in which case $\theta_1 = 0$). Together with the facts that $V_t^{\mathrm{A}}(\theta_t, \theta_0)$ decreases linearly in θ_t and that $V_t^{\Omega}(\theta_t, \theta_0)$ is decreasing and strictly convex in θ_t it follows that $V_t^{\Omega}(\theta_t, \theta_0) > V_t^{\mathrm{A}}(\theta_t, \theta_0)$ holds also for all $t > 0$ in this situation. Hence, $F = (\Omega, ...)$ must be optimal.

As an intermediate result, it follows that $F \in \mathcal{F}^D$ for all $\delta_S > \delta_S'$.

Step 3: We rule out the non-shipment deviation for $F = (\mathrm{A}, \Omega, \Omega, ...)$.

Remains to rule out the non-shipment deviation for the seller under the payment sequence $F = (\mathrm{A}, \Omega, \Omega, ...)$ (analogy to Lemma 1). First, let us derive the ex-ante expected payoffs for the sequence $F = (\mathrm{A}, \Omega, \Omega, ...)$. They are obtained from solving the following recursion for $V_0^{\mathrm{A}\Omega}$:

$$V_0^{\mathrm{A}\Omega} = (1-\theta_0)\overline{\pi}^{\mathrm{A}} + \delta_S\left[\gamma(1-\theta_0)V_1^{\mathrm{A}\Omega} + (1-\gamma(1-\theta_0))V_0^{\mathrm{A}\Omega}\right], \qquad V_1^{\mathrm{A}\Omega} = \frac{\overline{\pi}^{\Omega}}{1-\delta_S}.$$

The solution is:

$$\Pi^{\mathrm{A}\Omega} = \frac{(1-\theta_0)(\delta_S\gamma\overline{\pi}^{\Omega} + (1-\delta_S)\overline{\pi}^{\mathrm{A}})}{(1-\delta_S)(1-\delta_S(1-\gamma(1-\theta_0)))}.$$

A deviation by the seller by not procuring the product in the initial transaction on A-terms is ruled out if and only if:

$$-cQ^{\mathrm{A}} + \delta_S V_1^{\mathrm{A}\Omega} \geq \delta_S V_0^{\mathrm{A}\Omega} \quad \Leftrightarrow \quad \Gamma_1 \equiv \left(\frac{\delta_S\gamma}{1-\delta_B(1-\gamma)}\right)^{\frac{1}{\alpha}} - (1-\theta_0) \geq \frac{(1-\alpha)(1-\delta_S(1-\gamma(1-\theta_0)))}{\alpha\delta_S} \equiv \Gamma_2.$$

We want to derive parameter requirements such that $\Gamma_1 \geq \Gamma_2$ holds. First, note that $\partial\Gamma_2/\partial\alpha < 0$, $\partial^2\Gamma_2/\partial\alpha^2 > 0$, $\lim_{\alpha\to 0}\Gamma_2 = \infty$ and $\lim_{\alpha\to 1}\Gamma_2 = 0$. Second, note that $\partial\Gamma_1/\partial\alpha > 0$ and $\lim_{\alpha\to 0}\Gamma_1 = -(1-\theta_0)$. Hence there exists a unique $\tilde{\alpha}^o \in (0,1)$ such that $\Gamma_1 \geq \Gamma_2$ for all $\alpha > \tilde{\alpha}^o$ if and only if:

$$\lim_{\alpha\to 1}\Gamma_1 > 0 \quad \Leftrightarrow \quad \delta_S > \gamma^{-1}(1-\theta_0)(1-\delta_B(1-\gamma)) \equiv \tilde{\delta}_S^o.$$

We need to ensure that $\tilde{\delta}_S^o \in (0,1)$. This is the case if and only if:

$$\delta_B > \frac{1-\theta_0-\gamma}{(1-\theta_0)(1-\gamma)} \equiv \underline{\delta}_B \in (0,1). \tag{A.10}$$

We conclude that the non-shipment deviation under the sequence $F = (\mathrm{A}, \Omega, \Omega, ...)$ is ruled out whenever $\alpha > \tilde{\alpha}^o$, $\delta_S > \tilde{\delta}_S^o$ and $\delta_B > \underline{\delta}_B$ hold.

Step 4: Summary of the parameter constraints.

Let us summarize all the parameter requirements that we derived above and in Lemma 1 which allow us to conclude that $F \in \mathcal{F}^D$. Besides $\delta_B > \underline{\delta}_B$, the constraints are:

$$\begin{aligned} \alpha &> \max\{\tilde{\alpha}, \tilde{\alpha}^o\} \equiv \underline{\alpha} \in (0,1), \\ \delta_S &> \max\{\tilde{\delta}_S, \tilde{\delta}_S^o, \delta_S'\} \equiv \underline{\delta}_S \in (0,1). \ \blacksquare \end{aligned} \tag{A.11}$$

Proof of Corollary 1

We begin by deriving essential comparative statics of the ex-ante expected payoff functions. First, let us compare the limit properties with respect to the initial belief θ_0. Observe that $\lim_{\theta_0 \to 1} \Pi^{A\Omega} = \lim_{\theta_0 \to 1} \Pi^{A} = 0 < \lim_{\theta_0 \to 1} \Pi^{\Omega} = \lambda^{\frac{1}{\alpha}} \overline{\pi}^{\Omega}/(1-\delta_S)$. Moreover, we have:

$$\lim_{\theta_0 \to 0} \Pi^{A\Omega} = \frac{\gamma \delta_S \overline{\pi}^{\Omega} + (1-\delta_S)\overline{\pi}^{A}}{(1-\delta_S)(1-\delta_S(1-\gamma))}, \qquad \lim_{\theta_0 \to 0} \Pi^{A} = \frac{\overline{\pi}^{A}}{1-\delta_S}, \qquad \lim_{\theta_0 \to 0} \Pi^{\Omega} = \frac{\overline{\pi}^{\Omega}}{1-\delta_S},$$

for which holds $\lim_{\theta_0 \to 0} \Pi^{A} > \lim_{\theta_0 \to 0} \Pi^{A\Omega} > \lim_{\theta_0 \to 0} \Pi^{\Omega}$. Next, we derive essential functional properties of Π^{A}, $\Pi^{A\Omega}$, and Π^{Ω}. We get:

$$\frac{\partial \Pi^{A}}{\partial \theta_0} = -\frac{(1-\delta_S \gamma)\overline{\pi}^{A}}{(1-\delta_S)(1-\delta_S\gamma\theta_0)^2} < 0, \quad \frac{\partial \Pi^{A\Omega}}{\partial \theta_0} = -\frac{(1-\delta_S)\overline{\pi}^{A} + \delta_S \gamma \overline{\pi}^{\Omega}}{(1-\delta_S+\delta_S\gamma(1-\theta_0))^2} < 0,$$
$$\frac{\partial^2 \Pi^{A}}{\partial \theta_0^2} = -\frac{2\delta_S\gamma(1-\delta_S \gamma)\overline{\pi}^{A}}{(1-\delta_S)(1-\delta_S\gamma\theta_0)^3} < 0, \qquad \frac{\partial^2 \Pi^{A\Omega}}{\partial \theta_0^2} = -\frac{2\delta_S\gamma[(1-\delta_S)\overline{\pi}^{A} + \delta_S \gamma \overline{\pi}^{\Omega}]}{(1-\delta_S+\delta_S\gamma(1-\theta_0))^3} < 0.$$

Determining the signs of the derivatives of Π^{Ω} with respect to the initial belief is more involved. Let us define:

$$\Pi_t^{\Omega} \equiv \frac{(1-\lambda\delta_S)(1-\theta_0(1-\lambda^t))}{1-\delta_S(\theta_0+(1-\theta_0)\lambda)} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} \overline{\pi}^{\Omega},$$

where $\Pi^{\Omega} = \sum_{t=0}^{\infty} \Pi_t^{\Omega}$. It holds that:

$$\frac{\partial \Pi_t^{\Omega}}{\partial \theta_0} < 0 \quad \Leftrightarrow \quad (1-\lambda)\lambda^t(1-\delta_S\lambda-\delta_S\theta_0(1-\lambda)) + \alpha(1-\delta_S-\lambda^t(1-\delta_S\lambda))(1-\theta_0(1-\lambda^{t+1})) > 0,$$

implying that $\partial \Pi^{\Omega}/\partial \theta_0 < 0$. It turns out that Π^{Ω} is concave only under additional parameter restrictions. We get:

$$\frac{\partial^2 \Pi_t^{\Omega}}{\partial \theta_0^2} < 0 \quad \Leftrightarrow \quad K(t,\delta_S,\lambda,\theta_0,\alpha) \equiv \frac{1-\alpha}{\alpha}\Delta(t,\delta_S,\lambda,\theta_0) - 2\delta_S(1-\lambda)\left[E(t,\delta_S,\lambda,\theta_0) + \alpha Z(t,\delta_S,\lambda)\right] < 0, \tag{A.12}$$

$$\text{where} \quad \Delta(t,\delta_S,\lambda,\theta_0) \equiv \frac{(1-\delta_S\lambda-\delta_S\theta_0(1-\lambda))^2(1-\lambda)\lambda^t}{(1-\theta_0(1-\lambda^{t+1}))^2(1-\theta_0(1-\lambda^t))} > 0,$$
$$E(t,\delta_S,\lambda,\theta_0) \equiv \frac{(1-\delta_S\lambda-\delta_S\theta_0(1-\lambda))(1-\lambda)\lambda^t}{(1-\theta_0(1-\lambda^{t+1}))} > 0,$$
$$Z(t,\delta_S,\lambda) \equiv 1-\delta_S-\lambda^t(1-\delta_S\lambda).$$

Let $H(t,\delta_S,\lambda,\theta_0) \equiv E(t,\delta_S,\lambda,\theta_0) + \alpha Z(t,\delta_S,\lambda)$. Observe that $H > 0$ for all $\alpha \in (0,1)$ if we establish that $H\mid_{\alpha\to 1} > 0$ since Z is possibly negative, and $E > 0$. We get:

$$H\mid_{\alpha\to 1} > 0 \quad \Leftrightarrow \xi(t,\delta_S,\lambda) \equiv \quad 1-\lambda^{t+1}-\delta_S(1-\lambda^{t+2}) > 0.$$

Since $\partial\xi/\partial t > 0$, it is sufficient to check $\xi\mid_{t=0} > 0$. Rearranging the latter gives:

$$\lambda < \frac{1-\delta_S}{\delta_S} \equiv \overline{\lambda} > 0. \tag{A.13}$$

Consequently, under the assumption that $\lambda < \overline{\lambda}$, we have that K is decreasing in α and since $\lim_{\alpha\to 1} K = -2\delta_S(1-\lambda)H < 0$ and $\lim_{\alpha\to 0} K = \infty$ there must exist $\overline{\alpha} \in (0,1)$ such that $K < 0$ for all $\alpha > \overline{\alpha}$. We therefore conclude that Π^{Ω} is concave in θ_0 for all $\alpha > \overline{\alpha}$ and all $\lambda < \overline{\lambda}$.

Irrespective of the concavity of Π^{Ω}, from the above functional properties it follows that for sufficiently small (respectively high) values of θ_0, $F = (\mathrm{A}, ...)$ (respectively $F = (\Omega, ...)$) is payoff-maximizing for the seller. As established in the proof of Proposition 3, also observe that:

$$\Pi^{\mathrm{A}\Omega} > \Pi^{\mathrm{A}} \quad \Leftrightarrow \quad \theta_0 > \underline{\theta}_0 \in (0,1).$$

From this it directly follows that $(\Omega, ...)$ can only be optimal for $\theta_0 > \underline{\theta}_0$ as well. Clearly, due to the limit properties of the payoff functions for $\theta_0 \to 1$, only $(\Omega, ...)$ can be optimal in this case.

When, in addition, $\alpha > \overline{\alpha}$ and $\lambda < \overline{\lambda}$ hold and Π^{Ω} is strictly concave, it follows from the limit properties of the payoff functions and their first and second derivatives that there exists a unique $\overline{\theta}_0 \in (\underline{\theta}_0, 1)$ such that $\Pi^{\mathrm{A}\Omega} > \max\{\Pi^{\mathrm{A}}, \Pi^{\Omega}\}$ for all $\theta_0 \in (\underline{\theta}_0, \overline{\theta}_0)$ and $\Pi^{\Omega} > \max\{\Pi^{\mathrm{A}}, \Pi^{\mathrm{A}\Omega}\}$ for all $\theta_0 > \overline{\theta}_0$. ■

Proof of Corollary 2

First, note that I-terms cannot follow on A-terms because at the full information limit I-terms are dominated by Ω-terms. The reason is that when A-terms are used before the game reaches the full information limit and by playing Ω-terms instead of I-terms the seller can save the fixed costs of the insurance, m, in this case.

Second, note that I-terms cannot follow on Ω-terms. To see this, let us rewrite the belief under payment contract $j \in \{\Omega, \mathrm{I}\}$ for period $t+1$ as $\theta^j_{t+1} = \theta^j_t \lambda / (1 - \theta^j_t (1-\lambda))$. Note that θ^j_{t+1} is an increasing and strictly convex function in θ^j_t. Consequently, the incentive to employ insurance is largest in the initial period since it implies the largest informational gain from the insurer's screening activity. Hence, whenever trade credit insurance is used it will be employed in the initial transaction.

Note also, that insurance will not be used for more than the initial period. The reason is that in any further transaction with the same buyer the seller can benefit from the insurer's screening technology also under Ω-terms. However, by not using the insurance he can save the fixed insurance costs m in the subsequent periods.

Remains to establish that A-terms cannot follow on an initial period on I-terms. Since the value functions V^{I}_t and V^{Ω}_t are structurally equivalent, the comparative statics of V^{Ω}_t w.r.t. θ_t derived in Step 2 of the proof of Proposition 3 apply analogously to V^{I}_t. This directly implies that the seller will never find it optimal to switch to A-terms after an initial transaction on I-terms.

Consequently, the only sequence of payment contracts that can become dynamically optimal and includes insurance terms is $F = (\mathrm{I}, \Omega, \Omega, ...)$. The corresponding ex-ante expected payoffs are obtained from the following program:

$$\begin{aligned} V^{\mathrm{I}\Omega}_0 &= \pi^{\mathrm{I}}_0 + \delta_S \left(\Lambda^{\mathrm{I}}_0 V^{\mathrm{I}\Omega}_1 + (1 - \Lambda^{\mathrm{I}}_0) V^{\mathrm{I}\Omega}_0 \right), \\ \forall t > 0: \quad V^{\mathrm{I}\Omega}_t &= \pi^{\mathrm{I}}_t + m + \delta_S \left(\Lambda^{\mathrm{I}}_t V^{\mathrm{I}\Omega}_{t+1} + (1 - \Lambda^{\mathrm{I}}_t) V^{\mathrm{I}\Omega}_0 \right). \end{aligned} \tag{A.14}$$

Solving (A.14) for $V^{\mathrm{I}\Omega}_0$ by using the same steps as in the derivation of Π^{Ω} gives:

$$\Pi^{\mathrm{I}\Omega} = \frac{1 - \delta_S \lambda}{1 - \delta_S \lambda - \delta_S \theta^{\mathrm{I}}_0 (1-\lambda)} \left[-m + \overline{\pi}^{\Omega} \sum_{t=0}^{\infty} \delta^t_S (\Lambda^{\mathrm{I}}_t)^{\frac{1}{\alpha}} (1 - \theta^{\mathrm{I}}_0 (1 - \lambda^t)) \right]. \ ■$$

Proof of Corollary 3

As argued in the proof of the previous Corollary 2, the comparative statics of V_t^{Ω} w.r.t. θ_t also apply to V_t^{I}. As a consequence, we have that $\Pi^{\mathrm{I}\Omega}$ decreases monotonically in θ_0. Next, let us compare the limit properties of Π^{Ω} and $\Pi^{\mathrm{I}\Omega}$ w.r.t. θ_0. First, note that $\lim_{\theta_0 \to 0} \Pi^{\mathrm{I}\Omega} = -m + \overline{\pi}^{\Omega}/(1-\delta_S) < \lim_{\theta_0 \to 0} \Pi^{\Omega}$. Since both, Π^{Ω} and $\Pi^{\mathrm{I}\Omega}$ are monotonically decreasing and continuous in θ_0, whenever:

$$\begin{aligned} &\lim_{\theta_0 \to 1} \Pi^{\mathrm{I}\Omega} > \lim_{\theta_0 \to 1} \Pi^{\Omega} \\ \Leftrightarrow \quad & m < \overline{\pi}^{\Omega} \left[\frac{\lambda^{\frac{1}{\alpha}}(1-\delta_S\lambda - \delta_S\phi(1-\lambda))}{(1-\delta_S)(1-\delta_S\lambda)} - \sum_{t=0}^{\infty} \delta_S^t \left(\frac{1-\phi(1-\lambda^{t+1})}{1-\phi(1-\lambda^t)} \right)^{\frac{1}{\alpha}} (1-\phi(1-\lambda^t)) \right] \equiv \overline{m}, \end{aligned} \tag{A.15}$$

then there exists a $\hat{\theta}_0' \in (0,1)$ at which $\Pi^{\mathrm{I}\Omega} = \Pi^{\Omega}$, and $\Pi^{\mathrm{I}\Omega} > \Pi^{\Omega}$ if $\theta_0 > \hat{\theta}_0'$. Noting from Corollary 1 that for $\theta_0 \to 1$ the sequence $(\Omega, ...)$ payoff-dominates $(\mathrm{A}, ...)$ and $(\mathrm{A}, \Omega, \Omega, ...)$, we can infer that there must exist $\hat{\theta}_0 \in [\hat{\theta}_0', 1)$ such that for all $\theta_0 > \hat{\theta}_0$ we have that $\Pi^{\mathrm{I}\Omega} > \max\{\Pi^{\Omega}, \Pi^{\mathrm{A}\Omega}, \Pi^{\mathrm{A}}\}$. ■

A.2 Model extension: Private observability of revenue shocks

In this extension, we study the situation where, in any period, the buyer learns the realization of the revenue level r_t privately. We allow the buyer to make a non-verifiable report $\hat{r}_t$ of the revenue realization to the seller and adjust the revenue realization stage of the game as follows.

1. ***Revenue realization.*** The level of the revenue shifter $r_{t-1} \in \{0,1\}$ is realized and privately learned by the buyer. The buyer decides on a non-verifiable revenue report $\hat{r}_{t-1} \in \{0,1\}$ to the seller. The product shipped in the previous period generates revenue $R(Q_{t-1}, r_{t-1})$ to the buyer from the sale to final consumers.

Under A-terms, the buyer's report is irrelevant for optimal contract design. Since at the contracting and the payment stage both – buyer and seller – do not know the realized revenue level the report about it is irrelevant for contract design and relationship continuation. Consequently, the analysis does not change when compared to the main text.

Under Ω-terms, it may not be optimal for the seller to set the same transfers as in the public information case and condition the applicable transfer level on reported instead of on realized revenues. Such conditioning creates an incentive for the buyer to under-report strategically when revenues are high, i.e. to report $\hat{r}_t = 0$ in all periods. Consequently, as shown by Troya-Martinez (2013, 2017) the seller has two options. Either, he can propose a flat contract to the buyer in which the transfer does not condition on reported revenues. Alternatively, the contract may contain report-contingent transfers and ensure truthful reporting by punishing low reports with trade suspension. We investigate both cases in the following.

Report-dependent transfers with truthtelling incentivization under Ω-terms

We first consider the scenario where the seller offers report-contingent transfers to the buyer. For now, assume that the seller assigns the same transfers to reported revenues levels as those chosen for the respective levels in the public information case.[32] This implies that the prescribed transfer when reported revenues are high ($\hat{r}_t = 1$) is larger than when the revenue report is low ($\hat{r}_t = 0$). In this situation, the myopic buyer always reports low since it gives her larger stage payoffs also in the situation where contracts are enforced and deviation is not possible. For the patient buyer, on the one side it is never optimal to over-report when $r_t = 0$ since $T_t^{\Omega,h} > T_t^{\Omega,l}$ leads to immediate bankruptcy which is not optimal given positive continuation payoffs under truthtelling. Conversely, when $r_t = 1$ the buyer has an incentive to under-report, since the lower transfer when $\hat{r}_t = 0$ ensures her a higher stage payoff.

The seller can counter the under-reporting problem under private information by incentivizing the patient buyer to tell the truth (such that she reports $\hat{r}_t = r_t$ in all periods) by (temporarily) suspending trade when $\hat{r}_t = 0$. The length of trade suspension is chosen such that the buyer is indifferent between possible reports (cf. Troya-Martinez, 2017). As outlined above, the myopic buyer can never be incentivized to report high revenues truthfully. As a consequence, a high report of the buyer is a *credible signal* of her patient type effectuating an update of the seller's belief to $\theta_{t+1} = 0$ in the following period. Note that such signal enhances the buyer's continuation payoff only if the realized revenue is indeed high since the corresponding transfer would lead to her bankruptcy otherwise (this eliminates any incentive for strategic over-reporting).

Before we set up the seller's dynamic programming problem in which we incorporate the above observations, let us first derive the optimal stage contract with report-dependent transfers that ensures truthtelling. Compared to the public information case the payment probability is adjusted in order to account for the reporting behavior of buyers outlined above. In period t, the seller chooses $\{Q_t, T_t^{\Omega,h}, T_t^{\Omega,l}, T(l)\}$ to maximize the following stage payoff function:

$$\pi_t^{\Omega,s} = \delta_S \left[(1-\theta_t)\gamma T_t^{\Omega,h} + \left[(1-\gamma)(1-\theta_t) + \theta_t\lambda\right] T_t^{\Omega,l}\right] - cQ_t. \tag{A.16}$$

$T(l) \geq 0$ denotes the number of trade suspension periods following on a low revenue report. Evidently, trade suspension in the high revenue state reduces seller payoffs while not increasing the buyer's incentive to report truthfully. Hence, we do not need to further consider this possibility in the following.

While the maximization problem is subject to the same participation and liquidity constraints as in the public information case the buyer's incentive constraints are adjusted as follows:

$$u_t^l \equiv -T_t^{\Omega,l} + \frac{\delta_B^{T(l)+1}}{1-\delta_B}\left[\gamma R(Q_t) - ET_t^{\Omega}\right] \geq 0, \tag{IC$_{B,t}^{\Omega,l}$}$$

$$u_t^h \equiv -T_t^{\Omega,h} + \frac{\delta_B}{1-\delta_B}\left[\gamma R(Q_t) - ET_t^{\Omega}\right] \geq 0. \tag{IC$_{B,t}^{\Omega,h}$}$$

Finally, to ensure truthtelling we need $u_t^l = u_t^h$ as an additional constraint to the maximization problem.[33]

The derivation of the optimal equilibrium transfers follows the exact same steps as in Lemma 2, which applies one-to-one here. We can plug the resulting transfer payments $T_t^{\Omega,l} = 0$ and $T_t^{\Omega,h} = \delta_B\gamma/(1-\delta_B(1-\gamma))R(Q_t)$ into the truthtelling constraint, which implies that trade with

[32] We show below that the optimal transfers under private information are identical to those of the public information case.

[33] We assume that the trade relationship with the suspended buyer ends permanently when the seller decides to engage in a new trade relationship during periods of trade suspension.

a buyer is suspended permanently whenever $\hat{r}_t = 0$, i.e. $T(l) = \infty$. Note that this result is consistent with the analysis by Troya-Martinez (2017).

Using the equilibrium transfer payments the seller chooses the trade volume in period t by maximizing the following variant of (A.16):

$$Q_t^{\Omega,s} \equiv \arg\max_{Q_t} \delta_S(1-\theta_t)\mathcal{T}R(Q_t) - cQ_t.$$

The optimal trade volume $Q_t^{\Omega,s}$ and the corresponding stage game payoff $\pi_t^{\Omega,s}$ in the tth transaction with a buyer on open account terms can be calculated as:

$$Q_t^{\Omega,s} = \left(\frac{\delta_S\mathcal{T}(1-\theta_t)}{c}\right)^{\frac{1}{\alpha}}, \qquad \pi_t^{\Omega,s} = Q_t^{\Omega,s}\frac{c\alpha}{1-\alpha}.$$

We denote the stage payoffs at the full information limit by $\overline{\pi}^{\Omega,s}$ in the following.

For the remainder of the paragraph, suppose that the seller is restricted to Ω-terms with report-dependent transfers. Accounting for the possibility of type signalling under private information outlined above and conditional on the optimality of the trade suspension punishment, the seller's dynamic programming problem looks as follows in this situation:

$$V_0^{\Omega,s} = \pi_0^{\Omega,s} + \delta_S\left[(1-\gamma(1-\theta_0))V_0^{\Omega,s} + \gamma(1-\theta_0)V_1^{\Omega,s}\right],$$
$$V_1^{\Omega,s} = \overline{\pi}^{\Omega,s} + \delta_S\left((1-\gamma)V_0^{\Omega,s} + \gamma V_1^{\Omega,s}\right),$$

where re-matching (due to a low revenue report) occurs in equilibrium when the buyer is myopic, or, when the patient buyer faces a low revenue realization. Solving the problem for $V_0^{\Omega,s}$ gives the seller's ex-ante expected payoffs:

$$\Pi^{\Omega,s} = \frac{(1-\delta_S\gamma)\pi_0^{\Omega,s} + \delta_S\gamma(1-\theta_0)\overline{\pi}^{\Omega,s}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)}.$$

Report-independent transfers under Ω-terms

Alternatively to establishing truthtelling, the seller can ignore the buyer's revenue report and offer a contract with a transfer that depends only on the trade volume (a "flat contract" with regard to the reported revenue). In principle, the seller here has two options. First, he can set the transfer at a lower level such that the patient buyer does not suffer a risk of bankruptcy in either revenue state. Since this strategy is not profitable for the seller (it requires $T_t = 0$ in all periods) we will not consider it further. Alternatively, the seller can ignore the liquidity constraints and set the transfer such that the patient buyer's incentive constraint ($\text{IC}_{B,t}^{\Omega}$) binds with equality (($\text{PC}_{B,t}^{\Omega}$) is also satisfied in this case):

$$-T_t^{\Omega,f} + \frac{\delta_B}{1-\delta_B}[\gamma R(Q_t) - T_t^{\Omega,f}] \geq 0. \tag{$\text{IC}_{B,t}^{\Omega}$}$$

This implies $T_t^{\Omega,f} = \delta_B\gamma R(Q_t)$. Acknowledging this transfer, the seller chooses the trade volume in period t by maximizing the following stage payoff function:

$$\pi_t^{\Omega,f} = \delta_S\gamma\Lambda_t T_t^{\Omega,f} - cQ_t.$$

The payment probability is adjusted to $\gamma\Lambda_t$ to account for the fact that payment of the transfer $T_t^{\Omega,f}$ only occurs when revenues are high (no revenue is generated otherwise, and therefore no transfer is possible). In this situation, non-payment occurs only if the buyer is myopic and contracts are not enforced.

The optimal trade volume $Q_t^{\Omega,f}$ and the corresponding stage game payoffs with a buyer under belief θ_t can be calculated as:

$$Q_t^{\Omega,f} = \left(\frac{\delta_S \delta_B \gamma^2 \Lambda_t}{c}\right)^{\frac{1}{\alpha}}, \qquad \pi_t^{\Omega,f} = Q_t^{\Omega,f} \frac{c\alpha}{1-\alpha}.$$

We denote the stage payoffs at the full information limit by $\overline{\pi}^{\Omega,f}$ in the following.

For the remainder of the paragraph, suppose that the seller is restricted to Ω-terms with report-independent transfers. Compared to the main text, the seller's dynamic programming problem needs to be adjusted by the fact that the relationship survives from one transaction to the next only if the revenue realization is high. Hence, we have:

$$\forall t \geq 0: \quad V_t^{\Omega,f} = \pi_t^{\Omega,f} + \delta_S \left(\gamma \Lambda_t V_{t+1}^{\Omega,f} + (1-\gamma\Lambda_t) V_0^{\Omega,f}\right). \tag{A.17}$$

Rewriting (A.17) in steps analogous to the main text, we get:

$$V_t^{\Omega,f} = \frac{1}{1-\theta_0(1-\lambda^t)} \left[\overline{\pi}^{\Omega,f} \sum_{i=t}^{\infty} \delta_S^{i-t} \Lambda_i^{\frac{1}{\alpha}} (1-\theta_0(1-\lambda^i)) + \delta_S V_0^{\Omega,f} \left(\frac{(1-\gamma)(1-\theta_t)}{1-\delta_S\gamma} + \frac{\theta_t \lambda^t (1-\gamma\lambda)}{1-\delta_S\gamma\lambda}\right)\right]. \tag{A.18}$$

We can solve the initial period version of (A.18) for $V_0^{\Omega,f}$ to obtain the ex-ante expected payoffs:

$$\Pi^{\Omega,f} = \frac{(1-\lambda\delta_S\gamma)(1-\delta_S\gamma)}{(1-\delta_S\gamma(\theta_0+(1-\theta_0)\lambda))(1-\delta_S)} \overline{\pi}^{\Omega,f} \sum_{t=0}^{\infty} \delta_S^t \Lambda_t^{\frac{1}{\alpha}} (1-\theta_0(1-\lambda^t)).$$

Optimal contract design with private information

In the following, we analyze dynamically optimal payment contract choice with private information. For any belief $\theta_t \in (0,1)$, the seller may now want to choose either A-terms or Ω-terms with report-dependent or -independent transfers. We introduce the same notation for the value functions as in the proof of Proposition 3 to distinguish more explicitly between the seller's belief in period t, θ_t, and his initial belief θ_0. This gives:

$$\begin{aligned}
V_t^{\mathrm{A}}(\theta_t,\theta_0) &= (1-\theta_t)\overline{\pi}^{\mathrm{A}} + \delta_S \left[\gamma(1-\theta_t)V_{t+1}(0,\theta_0) + (1-\gamma(1-\theta_t))V_{t+1}(\theta_0,\theta_0)\right], \\
V_t^{\Omega,s}(\theta_t,\theta_0) &= \pi_t^{\Omega,s} + \delta_S \left[\gamma(1-\theta_t)V_{t+1}(0,\theta_0) + (1-\gamma(1-\theta_t))V_{t+1}(\theta_0,\theta_0)\right], \\
V_t^{\Omega,f}(\theta_t,\theta_0) &= \pi_t^{\Omega,f} + \delta_S \left[\gamma\Lambda_t V_{t+1}(\theta_{t+1}^{\Omega},\theta_0) + (1-\gamma\Lambda_t)V_{t+1}(\theta_0,\theta_0)\right].
\end{aligned} \tag{A.19}$$

While under A-terms and report-dependent transfers the belief is updated to $\theta_{t+1} = 0$ at the beginning of the following transaction, under report-independent transfers updating follows Bayes' rule and $\theta_{t+1} = \theta_{t+1}^{\Omega}$. A comparison of $V_t^{\Omega,s}(\theta_t,\theta_0)$ and $V_t^{\mathrm{A}}(\theta_t,\theta_0)$ reveals that A-terms payoff-dominate the usage of Ω-terms with report-dependent transfers and truthtelling incentivization. This directly leads to the following Lemma.

Lemma A.1. *Under private information, in any transaction a payoff-maximizing seller will request payment either on* A*-terms, or on* Ω*-terms with a revenue report-independent transfer*

$T_t^{\Omega,f}$. *Incentivizing the buyer to report the revenue level truthfully is never payoff-maximizing for the seller under* Ω*-terms.*

Proof Analysis in the text.

This leaves us with two potentially optimal payment strategies under private information. The following Lemma A.2 provides a unique condition that pins down optimal payment contract choice for any period in a trade relationship.

Lemma A.2. *There exists a unique belief level* $\theta^* \in (0,1)$ *such that it is optimal for the seller in period* t *(under belief* θ_t*) to conduct business on* A*-terms if and only if* $\theta_t < \theta^*$*, and to use* Ω*-terms with report-independent transfers otherwise. This implies that the DOSPC is* $F \in \{(\mathrm{A},...),(\Omega,..,\Omega,\mathrm{A},\mathrm{A},...)\}$*. There exist initial belief levels* θ_0 *such that either type of sequence can be optimal in equilibrium.*

Proof First, observe that:

$$\lim_{\theta_t\to 1} V_t^{\Omega,f}(\theta_t,\theta_0) = \lim_{\theta_0\to 1} V_t^{\Omega,f}(\theta_0,\theta_0) = \frac{(\delta_S\gamma\lambda)^{\frac{1}{\alpha}}\,\overline{\pi}^{\mathrm{A}}}{1-\delta_S} > \lim_{\theta_t\to 1} V_t^{\mathrm{A}}(\theta_t,\theta_0) = 0.$$

Moreover:

$$V_t^{\mathrm{A}}(0,\theta_0) = \frac{1}{1-\delta_S\gamma}\left[\overline{\pi}^{\mathrm{A}} + \delta_S(1-\gamma)V_{t+1}(\theta_0,\theta_0)\right], \quad \text{and} \tag{A.20}$$

$$V_t^{\Omega,f}(0,\theta_0) = \frac{1}{1-\delta_S\gamma}\left[(\delta_S\gamma)^{\frac{1}{\alpha}}\,\overline{\pi}^{\mathrm{A}} + \delta_S(1-\gamma)V_{t+1}(\theta_0,\theta_0)\right],$$

from which it is easy to infer that $V_t^{\mathrm{A}}(0,\theta_0) > V_t^{\Omega,f}(0,\theta_0)$ holds. Next, note from the proof of Proposition 3 that V_t^{A} decreases linearly in θ_t. Moreover, due to the analogous functional structure of $V_t^{\Omega,f}(\theta_t,\theta_0)$ in (A.19) and of $V_t^{\Omega}(\theta_t,\theta_0)$ in (A.7) it follows by the same line of argument as in the proof of Proposition 3 that $V_t^{\Omega,f}(\theta_t,\theta_0)$ decreases and is convex in θ_t.

As a consequence, we can conclude that there exists a unique $\theta^* \in (0,1)$ such that $V_t^{\Omega,f} > V_t^{\mathrm{A}}$ if and only if $\theta_t > \theta^*$. Note that θ^* is a function of θ_0. From the limiting properties derived above it follows that there always exist values $\theta_0 \in (0,1)$ such that both sequences, $(\mathrm{A},...)$ and $(\Omega,..,\Omega,\mathrm{A},\mathrm{A},...)$, can be part of an optimal equilibrium. In sequence $(\Omega,..,\Omega,\mathrm{A},\mathrm{A},...)$, the period in which payment terms transition to A-terms is the first for which $\theta_t < \theta^*$ holds. ■

Figure A.1 summarizes the results of Lemma A.2 graphically. While the level of θ^* at which the two value functions intersect is θ_0-specific, depending on whether $\theta_0 \lesseqgtr \theta^*$ holds Ω-terms will be used in the initial periods of a trade relationship or not. It follows from the proof of Lemma A.2 that when θ_0 is close enough to the full information limit the seller will employ A-terms throughout, while Ω-terms will be used in the initial transactions if the share of myopic buyers is sufficiently large.

The figure depicts the situation where $\theta_0 > \theta^*$. In this case, after initial usage of Ω-terms the seller switches to A-terms beginning with the first period t in which $\theta_t^{\Omega} < \theta^*$ holds. The bullet points on the value functions indicate the steps of the belief updating process. In the plotted example, the seller's payment contract choice switches from Ω- to A-terms in period $t = 2$ of the trade relationship.

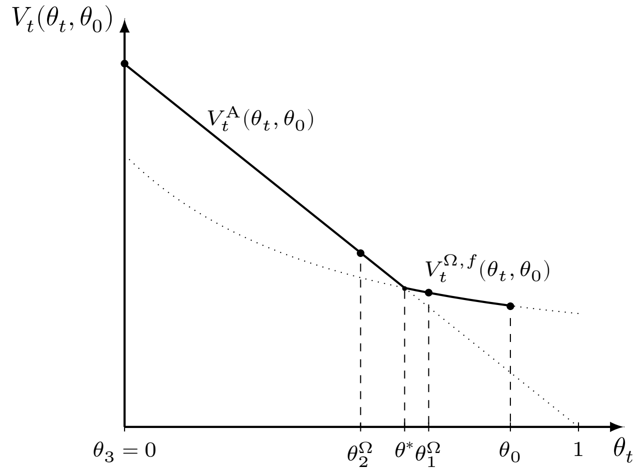

Figure A.1: *Value functions and belief evolution under private information when $\theta_0 > \theta^*$.*

Discussion

Lemma A.1 finds that only revenue report-independent transfers can be optimal under Ω-terms. This implies that the trade-off between relationship stability and stage payoff growth outlined in Section 3.3 applies also to the private information scenario. This is the case because truthtelling incentivization is never payoff-maximizing for the seller. Otherwise, patient buyers would have a tool available to signal their type which would imply immediate separation (similar to the situation under A-terms).

Note however, that different to the public information scenario, under private information it is optimal to employ the relationship stability-enhancing advantages of Ω-terms only temporarily on the learning path. When enough information about the buyer becomes available through repeated interaction and the full information limit is approximated sufficiently, A-terms payoff-dominate Ω-terms. Since under private information the optimal stage contract on Ω-terms is flat, a residual buyer bankruptcy risk also remains under these terms. As a consequence, the larger stage payoffs under A-terms at the full information limit also imply that these are overall more profitable in established relationships (which we show formally in Lemma A.2). Overall, our findings suggest that in established long-termed trade relationships it is more likely that sellers provide trade credit to their buyers if they can reliably observe the revenue realizations of buyers from final consumers.

A.3 Model extension: Generalization of the revenue shock distribution

In this Appendix, we generalize the model to account for revenue shocks of arbitrary size and assume that $r_t \in \{r^h, r^l\}$ with $r^h > r^l \geq 0$. As in the main text, we denote by $\gamma \in (0, 1)$ the probability that the revenue level is high, i.e. $r_t = r^h$. By $r_E = \gamma r^h + (1 - \gamma) r^l$ we denote the expected value of the revenue shifter. Assuming $r^l > 0$ makes the analysis of both, the cash in advance and the open account payment scenario, more involved. Under A-terms, depending on the parametrization of the revenue distribution, additional transfer strategies can be optimal for the seller and require to make further case distinctions in Lemma 1. Under Ω-terms, the seller now finds it optimal to request a non-zero transfer from the seller in the low revenue state which requires us to account for additional non-payment incentives of the buyer (implying adjustments to Lemma 2). We discuss the changes to the analysis of Section 3 in the following.

Cash in advance terms

In the main text scenario, whenever the patient buyer goes bankrupt under A-terms this happens when $r_t = 0$ in which case ($\text{LC}^{\text{A}}_{B,t}$) is not satisfied. While designing a contract that avoids this risk of buyer bankruptcy is never optimal there, the situation changes when $r^l > 0$ and requires to distinguish two cases.

On the one side, just as in the main text the seller may want to set the transfer to $T^{\text{A},h}_t = \delta_B R(Q_t, r_E)$ such that ($\text{PC}^{\text{A}}_{B,t}$) binds and extract all rents from the patient buyer. In this situation, the seller accepts that the buyer goes bankrupt when the low revenue state is realized. Alternatively, he can set the transfer to $T^{\text{A},l}_t = \delta_B R(Q_t, r^l) < T^{\text{A},h}_t$ such that the following liquidity constraint in the low revenue state binds:

$$\delta_B \mathcal{R}(Q_t, r^l) - T_t \geq 0, \tag{$\text{LC}^{\text{A},l}_{B,t}$}$$

which ensures that the trade relationship with the patient buyer is maintained in all revenue states.

Since revenue shocks are i.i.d. and the seller's learning about the buyer type does not depend on the transfer size, the seller's optimal decision between $T^{\text{A},h}_t$ and $T^{\text{A},l}_t$ does not vary over transactions. Hence, we can obtain the optimal transfer decision from comparing the seller's ex-ante expected payoffs when the transfer is fixed to either $T^{\text{A},h}$ or $T^{\text{A},l}$ for the entire relationship (the time index is dropped). In the following, we call the seller's choice $T^{\text{A}} \in \{T^{\text{A},l}, T^{\text{A},h}\}$ his *transfer strategy* under A-terms. For a given transfer strategy, the seller sets to trade volume by maximizing (1), and we denote the corresponding trade volumes by $Q^{\text{A},h}$ and $Q^{\text{A},l}$, respectively.

The following Lemma A.3 gives a unique condition on the revenue state distribution determining which of the two transfer levels is optimal for the seller and summarizes the corresponding trade volumes and profits.

Lemma A.3. *There exists a unique $\hat{\gamma} \in (0,1)$ such that setting the transfer to $T^{\text{A},h} = \delta_B R(Q^{\text{A},h}, r_E)$ in all transactions maximizes the seller's ex-ante expected payoffs if and only if $\gamma \geq \hat{\gamma}$, and setting it to $T^l = \delta_B R(Q^{\text{A},l}, r^l)$ in all transactions does so otherwise. Since any spot contract under* A*-terms is separating, trade volumes do not vary over time and are given as:*

$$Q^{\text{A}} = \begin{cases} (r_E \delta_B / c)^{\frac{1}{\alpha}} & \equiv Q^{\text{A},h} \quad \textit{if} \quad \gamma \geq \hat{\gamma}, \\ (r^l \delta_B / c)^{\frac{1}{\alpha}} & \equiv Q^{\text{A},l} \quad \textit{if} \quad \gamma < \hat{\gamma}. \end{cases} \tag{A.21}$$

The corresponding seller stage payoffs, conditional on contract acceptance, are:

$$\overline{\pi}^{\text{A}} = \begin{cases} (r_E \delta_B)^{\frac{1}{\alpha}} c^{\frac{\alpha-1}{\alpha}} \alpha/(1-\alpha) \equiv \overline{\pi}^{\text{A},h} & \textit{if} \quad \gamma \geq \hat{\gamma}, \\ (r^l \delta_B)^{\frac{1}{\alpha}} c^{\frac{\alpha-1}{\alpha}} \alpha/(1-\alpha) \equiv \overline{\pi}^{\text{A},l} & \textit{if} \quad \gamma < \hat{\gamma}. \end{cases} \tag{A.22}$$

Moreover, the seller's ex-ante expected payoffs are:

$$\Pi^{\mathrm{A}} = \begin{cases} \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A},h}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)} \equiv \Pi^{\mathrm{A},h} & \text{if} \quad \gamma \geq \hat{\gamma}, \\ \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A},l}}{(1-\delta_S)(1-\theta_0\delta_S)} \equiv \Pi^{\mathrm{A},l} & \text{if} \quad \gamma < \hat{\gamma}. \end{cases} \tag{A.23}$$

Proof The expressions in (A.21) and (A.22) are obtained from solving the maximization problem in (1) for the respective transfer strategy $T^{\mathrm{A}} \in \{T^{\mathrm{A},l}, T^{\mathrm{A},h}\}$. For the case where $T^{\mathrm{A}} = T^{\mathrm{A},h}$, the seller's ex-ante expected payoffs from conducting an infinite sequence of transactions on A-terms can be derived from solving the following dynamic programming problem for $V_0^{\mathrm{A},h}$:

$$\begin{aligned} V_0^{\mathrm{A},h} &= (1-\theta_0)\left[\overline{\pi}^{\mathrm{A},h} + \delta_S V_1^{\mathrm{A},h}\right] + \theta_0\delta_S V_0^{\mathrm{A},h}, \\ V_1^{\mathrm{A},h} &= \gamma[\overline{\pi}^{\mathrm{A},h} + \delta_S V_1^{\mathrm{A},h}] + (1-\gamma)V_0^{\mathrm{A},h}. \end{aligned}$$

Alternatively, in the situation where $T^{\mathrm{A}} = T^{\mathrm{A},l}$ the ex-ante expected payoffs are derived from the following problem:

$$\begin{aligned} V_0^{\mathrm{A},l} &= (1-\theta_0)\left[\overline{\pi}^{\mathrm{A},l} + \delta_S V_1^{\mathrm{A},l}\right] + \theta_0\delta_S V_0^{\mathrm{A},l}, \\ V_1^{\mathrm{A},l} &= \overline{\pi}^{\mathrm{A},l} + \delta_S V_1^{\mathrm{A},l}. \end{aligned}$$

The solutions to the respective programming problem are given in (A.23).

Next, note that the seller prefers to set $T^{\mathrm{A},h}$ instead of $T^{\mathrm{A},l}$ if and only if $\Delta\Pi \equiv \Pi^{\mathrm{A},h} - \Pi^{\mathrm{A},l} > 0$. Observing that $\partial\Delta\Pi/\partial\gamma > 0$, $\Delta\Pi \mid_{\lim_{\gamma\to 0}} < 0$ and $\Delta\Pi \mid_{\lim_{\gamma\to 1}} > 0$ establishes the existence of a unique probability level $\hat{\gamma} \in (0,1)$ such that the seller maximizes his ex-ante expected payoffs by setting $T^{\mathrm{A}} = T^{\mathrm{A},h}$ if $\gamma \geq \hat{\gamma}$ and by setting $T^{\mathrm{A}} = T^{\mathrm{A},l}$ otherwise. ■

The Lemma shows that even though setting the smaller transfer $T^{\mathrm{A},l}$ implies smaller optimal trade volumes ($Q^{\mathrm{A},l} < Q^{\mathrm{A},h}$) and, correspondingly, smaller stage payoffs ($\overline{\pi}^{\mathrm{A},l} < \overline{\pi}^{\mathrm{A},h}$) doing so can be optimal for the seller. When the probability of facing a low revenue state is sufficiently high (i.e., when $\gamma < \hat{\gamma}$) the seller prioritizes relationship stability over full rent-extraction from the buyer which he implements by choosing the smaller transfer level $T^{\mathrm{A},l}$.

Equivalently to Lemma 1, the following result rules out the non-shipment deviation by the seller. Since continuation payoffs depend on the chosen transfer strategy, each transfer scenario features distinct parameter thresholds to rule out the deviation. In Lemma A.4, we use the index $i \in \{l, h\}$ to refer to the low and high transfer strategy, respectively.

Lemma A.4. *Consider transfer strategy $i \in \{l, h\}$. Suppose that $\alpha > \tilde{\alpha}^i \in (0,1)$ holds. Then there exists a repeated game equilibrium that maximizes the seller's ex-ante expected payoffs under cash in advance terms, Π^{A}, for all $\delta_S \geq \tilde{\delta}_S^i \in (0,1)$.*

Proof At the Production and Shipment stage of any period the seller will not deviate from the contract if and only if:

$$-cQ^{\mathrm{A},i} + \delta_S V_1^{\mathrm{A},i} \geq \delta_S V_0^{\mathrm{A},i}, \qquad i = l, h. \tag{A.24}$$

Equation (A.24) follows from the same logic as (IC_S). Plugging explicit values for $V_0^{\mathrm{A},i}$ and $V_1^{\mathrm{A},i}$

into (A.24) and simplifying gives:

$$-cQ^{\mathrm{A},h} + \delta_S \frac{(1-\theta_0+\gamma\theta_0(1-\delta_S))\overline{\pi}^{\mathrm{A},h}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)} \geq \delta_S \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A},h}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)} \quad \text{for} \quad i = h, \tag{A.25}$$

$$\text{and} \quad -cQ^{\mathrm{A},l} + \delta_S \frac{\overline{\pi}^{\mathrm{A},l}}{1-\delta_S} \geq \delta_S \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A},l}}{(1-\delta_S)(1-\theta_0\delta_S)} \quad \text{for} \quad i = l. \tag{A.26}$$

Observing that $cQ^{\mathrm{A},i} = \overline{\pi}^{\mathrm{A},i}(1-\alpha)/\alpha$, $i = l, h$, we can simplify (A.25) to:

$$\delta_S \geq \frac{1-\alpha}{\gamma\theta_0} \equiv \tilde{\delta}_S^h.$$

For an equilibrium to exist we need to ensure that $\tilde{\delta}_S^h < 1$. This is the case whenever $\alpha > 1-\gamma\theta_0 \equiv \tilde{\alpha}^h \in (0,1)$ holds. In this situation, the non-production deviation of the seller can be ruled if he is patient enough, i.e. when $\delta_S \geq \tilde{\delta}_S^h$ holds. Moreover, we can simplify (A.26) to:

$$\delta_S \geq \frac{1-\alpha}{\theta_0} \equiv \tilde{\delta}_S^l,$$

and ensure that $\tilde{\delta}_S^l < 1$ by imposing that $\alpha > 1-\theta_0 \equiv \tilde{\alpha}^l \in (0,1)$ holds. ■

Under the conditions of Lemmas A.3 and A.4, Proposition 1 applies analogously for both transfer strategies discussed in this extension.

Open account terms

The seller's set of participation, liquidity, and incentive constraints remains structurally fully equivalent to the expressions in the main text. As a consequence, the pooling nature of the optimal spot contract – and hence the belief formation and updating process – remain the same. The size of revenue state-contingent transfers and thus the optimal trade volumes change, however. We summarize the principal changes under the generalized revenue shock distribution in the following Lemma A.5. It is the equivalent to Lemma 2 and ensures that the buyer behaves according to the strategy profile, while maximizing the seller's stage game payoffs.

Lemma A.5. *Suppose that $\delta_B \geq r^l/r_E \in (0,1)$. Then under Ω-terms, the seller sets transfers $T_t^{\Omega,l} = R(Q_t, r^l)$ and $T_t^{\Omega,h} = \delta_B\gamma/(1-\delta_B(1-\gamma))R(Q_t, r^h)$. Thereby, he rules out the buyer bankruptcy risk, makes the patient buyer indifferent between paying and not paying the agreed upon transfer in any revenue state and maximizes his own payoffs.*

Proof The proof of Lemma 2 applies with the following modifications. First, for $(\mathrm{LC}_{B,t}^{\Omega,l})$ to bind we set $T_t^{\Omega,l} = R(Q_t, r^l) > 0$. Second, to ensure that $T_t^{\Omega,h} \geq T_t^{\Omega,l}$ holds (which is used to incentivize buyer payment in any revenue state) we plug the explicit transfer levels into the expression which – after simplification – gives $\delta_B \geq r^l/r_E$. ■

In contrast to the case in main text, the generalized revenue shock distribution additionally requires that the patient buyer has a discount factor above a positive threshold level, i.e. $\delta_B \geq r^l/r_E$. This accounts for the additional non-payment deviation that becomes available to the buyer when $T^{\Omega,l} > 0$.

Acknowledging the results of Lemma A.5, the seller chooses the trade volume in period t by maximizing:

$$Q_t^\Omega \equiv \arg\max_{Q_t} \delta_S \Lambda_t \left[\frac{\delta_B \gamma^2}{1-\delta_B(1-\gamma)} R(Q_t, r^h) + (1-\gamma) R(Q_t, r^l) \right] - cQ_t.$$

The optimal trade volume Q_t^Ω and the corresponding stage game payoff π_t^Ω in the tth transaction with a buyer on open account terms can be calculated as:

$$Q_t^\Omega = \left(\frac{\delta_S \mathcal{T}'}{c} \Lambda_t \right)^{\frac{1}{\alpha}}, \qquad \pi_t^\Omega = Q_t^\Omega \frac{c\alpha}{1-\alpha}, \qquad \text{where} \quad \mathcal{T}' = \frac{\delta_B \gamma^2}{1-\delta_B(1-\gamma)} r^h + (1-\gamma) r^l.$$

The derivation of the seller's ex-ante expected payoffs is fully analogous to the main text. Moreover, Proposition 2 applies analogously.

A.4 Model extension: Positive buyer outside option

In this Appendix, we extend the analysis of section 3 to the situation where the buyer has a constant per-period outside option $\omega > 0$ that she receives when deciding not to engage in trade with the seller. Consistent with the strategy profile outlined in the main text, we assume that the seller ends the trade relationship permanently whenever the buyer decides to take the outside option instead of engaging in trade. After outlining all the differences to the analysis of the main text for the situation when $\omega > 0$ we summarize our findings in Proposition A.1 at the end of this Appendix.

Cash in advance terms

First, consider the case where $F = (\text{A}, ...)$. With the outside option available, the participation constraint of a buyer of type $j \in \{M, B\}$ in period t is:

$$\delta_j \mathcal{R}(Q_t, r_E) - T_t \geq \omega. \tag{PC$_{j,t}^{\text{A},\omega}$}$$

By the same logic as in the case of the main text where $\omega = 0$ the myopic buyer's participation constraint, (PC$_{M,t}^{\text{A},\omega}$), cannot be fulfilled for any $T_t > 0$. Consequently, the myopic buyer will never accept any contract on A-terms and the seller offers a *separating contract* that only a patient buyer accepts. Buyer liquidity constraints are unaffected by the size of the outside option.

As a consequence, the seller sets the transfer to $T_t^{\text{A},\omega} = \delta_B \mathcal{R}(Q_t, \gamma) - \omega$ such that (PC$_{B,t}^{\text{A},\omega}$) binds and extract the maximal amount of rents from the patient buyer. Acknowledging this transfer strategy, the seller's trade volume choice solves the following maximization problem:

$$Q_t^{\text{A},\omega} \equiv \arg\max_{Q_t} \pi_t^{\text{A},\omega} = T_t^{\text{A},\omega} - cQ_t, \tag{A.27}$$

which results in the following trade volume and stage payoffs:

$$Q^{\text{A},\omega} = \left(\frac{\gamma \delta_B}{c} \right)^{\frac{1}{\alpha}}, \qquad \overline{\pi}^{\text{A},\omega} \equiv \pi_t^{\text{A},\omega} = Q^{\text{A},\omega} \frac{c\alpha}{1-\alpha} - \omega.$$

Since trade volume $Q^{\text{A},\omega}$ and stage payoffs $\overline{\pi}^{\text{A},\omega} = \overline{\pi}^{\text{A}} - \omega$ do not vary with belief θ_t a necessary and sufficient condition for seller participation in the trade relationship is $\omega < \overline{\pi}^{\text{A}}$ who otherwise would refrain from engaging in trade altogether. For the following, we assume that this condition holds. The dynamic programming problem is structurally identical to the one derived in the main text. The seller's ex-ante expected payoffs are adapted as follows:

$$\Pi^{\mathrm{A},\omega} = \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A},\omega}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)}.$$

Equivalently to Lemma 1, the non-shipment deviation of the seller can be ruled out as summarized in the following Lemma A.6. For notational convenience we assume that $\omega \equiv w\overline{\pi}^{\mathrm{A}}$ in the following, where $w \in [0,1)$ needs to hold to satisfy seller trade participation.

Lemma A.6. *Suppose that $\alpha > \tilde{\alpha}^{\omega} \in (0,1)$ and $\omega \in [0, \overline{\pi}^{\mathrm{A}}]$. Then there exists a repeated game equilibrium that maximizes the seller's ex-ante expected payoffs under cash in advance terms, $\Pi^{\mathrm{A},\omega}$, for all $\delta_S \geq \tilde{\delta}_S^{\omega} \in (0,1)$.*

Proof Equivalently to the proof of Lemma 1, at the Production and Shipment stage (6) of any period the seller will not deviate from the contract if and only if:

$$-cQ^{\mathrm{A},\omega} + \delta_S \frac{(1-\theta_0+\gamma\theta_0(1-\delta_S))\overline{\pi}^{\mathrm{A},\omega}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)} \geq \delta_S \frac{(1-\theta_0)\overline{\pi}^{\mathrm{A},\omega}}{(1-\delta_S)(1-\gamma\theta_0\delta_S)}. \tag{A.28}$$

Observing that $cQ^{\mathrm{A},\omega} = \alpha^{-1}(1-\alpha)(1+w)\overline{\pi}^{\mathrm{A},\omega}$ we can simplify (A.28) to:

$$\delta_S \geq \frac{1-\alpha}{\gamma\theta_0}\frac{1+w}{1+(1-\alpha)w} \equiv \tilde{\delta}_S^{\omega}.$$

For an equilibrium to exist we need to ensure that $\tilde{\delta}_S^{\omega} < 1$. This is the case whenever $\alpha > (1-\gamma\theta_0)(1+w)/(1+w(1-\gamma\theta_0)) \equiv \tilde{\alpha}^{\omega} \in (0,1)$ holds. In this situation, the non-shipment deviation of the seller can be ruled if he is patient enough, i.e. when $\delta_S \geq \tilde{\delta}_S^{\omega}$ holds. ■

Note that $\partial\tilde{\delta}_S^{\omega}/\partial w > 0$, i.e. the minimum patience level of the seller necessary to sustain trade increases in the buyer's outside option. The reason is that with a higher outside option, the buyer only participates in trade when receiving a larger share of the revenue making it less attractive for the seller to obey the contract and indeed ship the product to the buyer. Asides, the results of Proposition 1 and the corresponding discussion hold analogously for the case where $\omega > 0$.

Open account terms

Next, consider the case where $F = (\Omega, ...)$. In the presence of the buyer's outside option her participation constraints become:

$$\gamma R(Q_t) - ET_t^{\Omega} \geq \omega, \tag{PC$^{\Omega,\omega}_{B,t}$}$$

$$\gamma R(Q_t) - \lambda ET_t^{\Omega} \geq \omega, \tag{PC$^{\Omega,\omega}_{M,t}$}$$

where ($\mathrm{PC}^{\Omega,\omega}_{B,t}$) is the participation constraint of the patient buyer and ($\mathrm{PC}^{\Omega,\omega}_{M,t}$) that of the myopic buyer, respectively. Conditional on the outside option not being too large (we derive an explicit constraint below), the screening properties of open account payment terms and the belief updating process remain unaffected when compared to the main text. As under A-terms, the liquidity constraints are unaffected in the presence of the outside option. However, the buyer's incentive constraints that ensure the payment of the transfer must be adapted in order to account for the outside option:

$$-T_t^{\Omega,l} + \frac{\delta_B}{1-\delta_B}[\gamma R(Q_t) - ET_t^{\Omega}] \geq \frac{\omega}{1-\delta_B}, \qquad (\text{IC}_{B,t}^{\Omega,l,\omega})$$

$$-T_t^{\Omega,h} + \frac{\delta_B}{1-\delta_B}[\gamma R(Q_t) - ET_t^{\Omega}] \geq \frac{\omega}{1-\delta_B}. \qquad (\text{IC}_{B,t}^{\Omega,h,\omega})$$

Equivalently to Lemma 2, the following Lemma A.7 derives the seller's optimal transfer strategy when $\omega \geq 0$. The Lemma also shows that an additional constraint on the size of the outside option is required to ensure seller participation in the trade relationship.

Lemma A.7. *Under Ω-terms, the seller participates in the trade relationship for all beliefs $\theta_t \in [0, \theta_0]$ if $\omega \in [0, \omega']$ and sets transfers $T_t^{\Omega,l,\omega} = 0$ and $T_t^{\Omega,h,\omega} = [\delta_B \gamma R(Q_t) - \omega]/(1-\delta_B(1-\gamma))$ in this situation. Thereby, he rules out the buyer bankruptcy risk, makes the patient buyer indifferent between paying and not paying the agreed upon transfer in any revenue state and maximizes his own payoffs.*

Proof As in Lemma 2, we require ($\text{LC}_{B,t}^{\Omega,l}$) to bind and set $T_t^{\Omega,l,\omega} = 0$. This allows us to rewrite ($\text{PC}_{B,t}^{\Omega,\omega}$) and ($\text{IC}_{B,t}^{\Omega,h,\omega}$) as:

$$T_t^{\Omega,h} \leq R(Q_t) - \frac{\omega}{\gamma} \equiv T^*, \qquad (\text{PC}_{B,t}^{\Omega,\omega})$$

$$T_t^{\Omega,h} \leq \frac{\delta_B \gamma R(Q_t) - \omega}{1-\delta_B(1-\gamma)} \equiv T_t^{\Omega,h,\omega}. \qquad (\text{IC}_{B,t}^{\Omega,h,\omega})$$

Note that ($\text{IC}_{B,t}^{\Omega,h,\omega}$) binds whenever $T^* > T_t^{\Omega,h,\omega} \Leftrightarrow \omega < \gamma R(Q_t)/(1-\gamma)$ holds. Seller participation in trade requires $T_t^{\Omega,h} > 0$ (he would make a loss otherwise). In this context, it is also necessary that $T^* > 0 \Leftrightarrow \omega < \gamma R(Q_t)$ holds, which ensures that $T^* > T_t^{\Omega,h,\omega}$ on the equilibrium path. Consequently, ($\text{IC}_{B,t}^{\Omega,h,\omega}$) is indeed the binding constraint and $T_t^{\Omega,h} = T_t^{\Omega,h,\omega}$ in equilibrium.

Acknowledging the equilibrium transfers derived above, the seller sets $Q_t^{\Omega,\omega}$ to maximize:

$$Q_t^{\Omega,\omega} \equiv \arg\max_{Q_t} \pi_t^{\Omega,\omega} = \delta_S \Lambda_t \gamma T_t^{\Omega,h,\omega} - cQ_t.$$

This gives $Q_t^{\Omega,\omega} = Q_t^{\Omega}$ and:

$$\pi_t^{\Omega,\omega} = \pi_t^{\Omega} - \frac{\delta_S \Lambda_t \gamma}{1-\delta_B(1-\gamma)}\omega.$$

To achieve comparability to the main text outcomes, our aim is to constrain ω such that the seller finds it profitable to trade with the buyer in every period (i.e. for every belief θ_t). This is the case if and only if:

$$\forall t \geq 0: \quad \pi_t^{\Omega,\omega} > 0 \quad \Leftrightarrow \quad \omega < \delta_B \gamma R(Q_t^{\Omega,\omega}) - \frac{1-\delta_B(1-\gamma)}{\delta_S \Lambda_t \gamma} cQ_t^{\Omega,\omega} \equiv \tilde{\omega}.$$

Since $\tilde{\omega} \in (0, \gamma R(Q_t^{\Omega,\omega}))$ and $\partial\tilde{\omega}/\partial\theta_t < 0$ a necessary and sufficient constraint to ensure seller participation in all periods is $\omega < \tilde{\omega}|_{t=0} \equiv \omega'$. ■

The seller's ex-ante expected payoffs are derived from a programming problem that is fully analogous to the main text and are given as:

$$\Pi^{\Omega,\omega} = \frac{1-\delta_S\lambda}{1-\delta_S\lambda-\delta_S\theta_0(1-\lambda)}\overline{\pi}^{\Omega,\omega}\sum_{t=0}^{\infty}\delta_S^t\Lambda_t^{\frac{1}{\alpha}}(1-\theta_0(1-\lambda^t)),$$

where $\overline{\pi}^{\Omega,\omega} = \overline{\pi}^{\Omega} - \delta_S\gamma\omega/(1-\delta_B(1-\gamma))$. The results of Proposition 2 and the corresponding discussion hold analogously for the case where $\omega > 0$.

We finish the discussion of the non-zero buyer outside option by summarizing the results of the model extension in the following Proposition.

Proposition A.1. *Suppose that instead to engaging in trade with the seller the buyer can decide to obtain a per-period outside option $\omega \in [0, \omega']$. A larger outside option allows the buyer to keep a larger revenue share in every period due to smaller equilibrium transfer levels and to realize larger transaction payoffs. At the same time, the seller's learning process about the buyer's type and relationship stability remain unaffected which reinforces the importance of the trade-offs identified in section 3.3.*

Proof Note that $\overline{\pi}^{\mathrm{A}} > \tilde{\omega}$, implying that $\omega < \omega'$ is a sufficient constraint on the outside option for both, Lemma A.6 and A.7 to be applicable. The remaining points follow from the discussion in the text above. ■

A.5 Model extension: Court usage and relationship stability

In this Appendix, we investigate the situation where the seller can observe when institutions (i.e. courts) are used to enforce contract compliance by the buyer. Since under A-terms only patient buyers accept the stage contract who – by construction – always comply with the contract terms, the analysis will not be affected in this payment scenario.

The situation changes under Ω-terms, however. While the buyer's participation and incentive constraints remain unvaried and therefore Lemma 2 applicable, the updating process of the seller's belief θ_t, trade volumes, stage payoffs, and the corresponding dynamic programming problem are subject to change. At the end of the first transaction with a buyer, the seller will know with certainty whether he is in a match with a patient or myopic buyer. The reason is that whenever a transaction with a myopic buyer is successful, it must be the case that buyer payment is enforced by court (she would never pay voluntarily). Contrarily, non-payment by the buyer will only occur if the buyer is myopic.

Hence, whenever an initial transaction is successful without the usage of courts (which happens if and only if the buyer is patient) the seller updates his belief from $\theta_0 = \widehat{\theta}$ to $\theta_1 = 0$. Correspondingly, trade volumes and stage payoffs grow from Q_0^{Ω} and π_0^{Ω} in the first transaction to Q^{Ω} and $\overline{\pi}^{\Omega}$ in the second transaction, respectively. Consistent with the findings by Macaulay (1963), we assume in the following that the seller discontinues the trade relationship once courts are used to enforce the transfer payment by the buyer. This gives rise to the following dynamic programming problem for the seller:

$$\begin{aligned} V_0^{\Omega,c} &= \pi_0^{\Omega} + \delta_S\left[(1-\theta_0)V_1^{\Omega,c} + \theta_0 V_0^{\Omega,c}\right], \\ V_1^{\Omega,c} &= \overline{\pi}^{\Omega} + \delta_S V_1^{\Omega,c}, \end{aligned}$$

which we can solve for $V_0^{\Omega,c}$ to obtain the seller's ex-ante expected payoffs:

$$\Pi^{\Omega,c} = \frac{\overline{\pi}^{\Omega}}{1-\delta_S} - \frac{\overline{\pi}^{\Omega} - \pi_0^{\Omega}}{1-\delta_S\theta_0}.$$

While under the varied model assumptions the belief updating process by the seller is the same under A- and Ω-terms and all information about the buyer is revealed until the end of the initial transaction, the qualitative predictions on trade volume growth and relationship stability of the main text remain valid. Since also under the varied assumptions the stage contract under Ω-terms cannot separate buyer types, just as in our baseline model, we see trade volume growth over time (while in contrast, trade volumes on A-terms do not vary over transactions). However, a difference is that due to the additional observability of court usage, the trade volume at the full information limit is reached already after the initial transaction.

Moreover, just as in the main text scenario the probability of relationship failure in any period is larger under A-terms than it is under Ω-terms. Under Ω-terms, a relationship fails after the initial transaction if and only if the buyer is myopic. Under A-terms, relationship breakdown additionally occurs when the patient buyer suffers bankruptcy (which does not occur under Ω-terms in equilibrium). Summing up, we find that our main results are qualitatively robust to assuming that the business relationship dies whenever courts are used to enforce the stage contract.